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ABSTRACT

This document focuses on research on the relation between children's use and understanding of counting as a symbol system for number and the development of number conservation concepts. An underlying assumption is that a better match between mathematics curricula and the educational needs of children will be facilitated by learning more about the factors which influence the formation of numerical skills. The research completed is presented in six sections; however section one containing overview articles has been removed from this document due to copyright restrictions. Section two reports on research on counting/number conservation relations. The third portion concerns developmental changes in children's conceptions of conventional counting properties. Section four presents studies of developmental changes in children's use of counting to represent numerical information. Section five discusses research on number concepts in Papua, New Guinea on topics other than conservation. The final portion presents planned research. Also included is a list of conference presentations, articles, chapters, and manuscripts disseminated during the funding period, and a list of additional research proposed during the support time of the projects.
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Counting and Number Conservation: Their Developments and Interrelations

Final Report

Prepared for National Institute of Education

Geoffrey B. Saxe

Principal Investigator

Project Number: NIE-G-78-0076

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Introduction

Counting and Number Conservation: Their Developments and Interrelations

Geoffrey B. Saxe

Principal Investigator

The primary focus of the research completed over the past two years has been the relation between children's use and understanding of counting as a symbol system for number and the development of number conservation concepts. An assumption underlying all of the research is that a better match between mathematics curricula and the educational needs of children will be facilitated by learning more about the factors which influence the formation of numerical skills.

The studies presented in this report follow the general outline of the original proposal, although the proposed studies were often modified on the basis of additional pilot testing. In addition, new and supplementary studies were designed to test additional hypotheses derived from or related to the proposed research. A variety of methodologies were employed in the supplemental work. Some supplemental work involved further analysis of data collected with atypical populations ("learning disabled"); other supplemental work involved studying number concepts in remote groups in Papua New Guinea (including counting/number conservation relations); still other work involved designing additional studies with New York City children. A list of papers completed or in progress during the funding period are listed under "Dissemination".

The research completed is presented in six sections. The first section consists of general overview articles which describe a programmatic

framework for the research and summarize some of the empirical findings presented in subsequent sections.

The second section reports research on counting/number conservation relations. Several of these manuscripts report findings with normal populations, others report comparative findings with Papua New Guinea populations, and still others with atypical populations in the United States. The comparative approach makes it possible to evaluate the adequacy of several models of counting/number conservation relations, an evaluation which would not be possible had we limited ourselves to studying normal Western children.

The third section concerns developmental changes in children's conceptions of conventional properties of counting. One study explores the character of these changes with monolingual children. Another demonstrates that bilingual children, having experience with multiple sign systems, achieve a mature understanding of conventional properties of counting prior to their monolingual peers.

In the fourth section, studies are presented on developmental changes in children's use of counting to represent numerical information. One study concerns the changing function of gesture in children's counting. Another study concerns the way in which the spatial configuration of a set of elements facilitates young children's ability to represent cardinal number. Another study concerns children's reflections about their counting accuracy across situations in which it is more or less difficult to achieve an accurate count. In addition, a coding manual for counting behavior is

is presented which we developed as the result of a large scale pilot study with young children.

In the fifth section, research is presented which was conducted on number concepts in Papua New Guinea which, unlike the New Guinea research presented in Section II, is concerned with concepts other than conservation. One study concerns the emergence of sophisticated logical concepts among Ponam children who live in the Manus area of Papua New Guinea. Other reports concern recent research with the Oksapmin of Papua New Guinea. The Oksapmin are a recently contacted group who use a 27 body part counting system for number. One study concerns developmental changes in Oksapmin children's use of the system to measure continuous quantities. The other two concern Oksapmin's use of the system in arithmetic computations. One of these compares the way in which schooled and non-schooled children use their bodies to perform arithmetic computations. The other concerns the way in which contact with economic exchange has influenced adults' use of their bodies to perform arithmetic computations.

In the sixth and final section, planned research is presented. The planned research will pursue three related areas in the development of numerical competencies. One area concerns the extent to which bilingual children show an advantage over their monolingual peers in numeration concepts (see Section III for suggestive findings). Another area concerns the way in which the early emergence of counting activities are facilitated in mother-child interaction, and the way in which these interactions differ over sub-cultural groups. A third direction concerns the developmental changes in children's ability to monitor their counting and computational activities.

DISSEMINATION DURING FUNDING PERIOD

Papers pending review at professional conferences

Saxe, G. B. The changing form of numerical thought as a function of contact with currency among the Oksapmin of Papua New Guinea. Submitted to 1981 Biennial meeting of the Society for Research in Child Development.

Saxe, G. B. Social change and cognitive growth: The invention of body part arithmetic algorithms among Oksapmin children in Papua New Guinea. Submitted to 1981 Biennial meeting of the Society for Research in Child Development.

Papers presented at professional conferences

Saxe, G. B. The origins of number symbolization in the child. Paper presented at workshop on children's mathematical cognition. Learning Research and Development Center. University of Pittsburgh, October, 1978.

Saxe, G. B. When fourth can precede second: A developmental analysis of a birth order numerational system among an island population in Papua New Guinea. Paper presented at 1979 Biennial meeting of the Society for Research in Child Development. San Francisco, California.

Saxe, G. B. Body parts as numerals: A comparative analysis of numeration in a remote village population in Papua New Guinea. Paper presented at 1979 Biennial meeting of Society for Research in Child Development. San Francisco, California.

Saxe, G. B., and Cohen, W. Figurative and operative aspects of number representation. Paper presented at Eastern Regional meeting of Society for Research in Child Development. Alexandria, Virginia, 1980.

Saxe, G. B., Schonfeld, I., & Segal, C. Cognition about counting. Paper presented at ninth annual symposium of the Jean Piaget Society. Philadelphia, May, 1979.

Articles, chapters and manuscripts prepared or in preparation

Saxe, G. B. Culture and the development of numerical cognition: Studies among the Oksapmin of Papua New Guinea. To appear in C. J. Brainerd (Ed.) Children's Logical and Mathematical Cognition, Springer-Verlag, 1981, in preparation.

Saxe, G. B. Number symbols and number operations: Their developments and interrelations. To appear in J. Johnston (Ed.) Topics in Language Disorders, 1981, in preparation

Articles, chapters and manuscripts prepared or in preparation (cont)

- Saxe, G. B. When fourth can precede second: A cross-cultural analysis of children's understanding of a birth order numerational system among an island population in Papua New Guinea. Journal of Cross-cultural Psychology, 1981.
- Saxe, G. B. Body parts as numerals: A developmental analysis of numeration among a village population in Papua New Guinea. Child Development, 1981.
- Saxe, G. B. Developmental relations between notational counting and number conservation. Child Development, 1979, 50, 180-187.
- Saxe, G. B. Children's counting: The early formation of numerical symbols. In Wolf, D. (Ed.) New Directions in Developmental Psychology, 1979, Vol. 1 (3), San Francisco: Jossey-Bass.
- Saxe, G. B. A comparative analysis of the acquisition of numeration: Studies from Papua New Guinea. Laboratory of Comparative Human Cognition Quarterly, 1979, Vol. 1, No. 3.
- Saxe, G. B., & Shaheen, S. Piagetian theory and the atypical case: An analysis of the developmental Gerstmann syndrome. Journal of Learning Disabilities, 1981.
- Saxe, G. B., & Posner, J. The development of numerical cognition: Cross-cultural perspectives. To appear in H. Ginsburg (Ed.) The Development of Mathematical Thinking. New York: Academic Press, 1981.
- Saxe, G. B., Cohen, W., & Rindskopf, D. Figurative and Operative Aspects of Number Representation. CUNY, 1980.
- Saxe, G. B., & Cohen, W. Empirical Operations and Logical Necessities: The role of counting in developing number conservation concepts. CUNY, 1980.
- Saxe, G. B., & Lancy, D. F. Culture, Counting and Number Conservation, CUNY, 1980.
- Saxe, G. B., & Sicilian, S. Children's Interpretations of their Counting Accuracy: A developmental analysis. CUNY, 1980.
- Saxe, G. B., & Kaplan, R. Gesture in Early Counting: A developmental analysis. CUNY, 1980.
- Saxe, G. B., Cohen, W., & Kaplan, R. A Coding Manual for Young Children's Counting Behavior. CUNY, 1979.
- Sicilian, S., & Saxe, G. B. The Role of Figurative Supports in Early Counting Behavior. CUNY, 1980.
- Saxe, G. B., Sicilian S., & Schonfeld, I. S. Correspondences and conventions: A developmental analysis of formal conceptions of counting. New York, 1979.

Articles, chapters and manuscripts prepared or in preparation (cont)

Saxe, G. B. The changing form of numerical thought as a function of contact with currency among the Oksapmin of Papua New Guinea. Manuscript in preparation.

Saxe, G. B. Social change and cognitive growth: The invention of body part arithmetic algorithms among Oksapmin children in Papua New Guinea. Manuscript in preparation

Saxe, G. B., & Moylan, T. Body parts as a standard measure: The development of measurement operations among the Oksapmin of Papua New Guinea. Manuscript in preparation.

Saxe, G. B., & Cohen, W. The use of counting to reproduce sets: The effects of set size and spatial configuration. Manuscript in preparation.

ADDITIONAL RESEARCH ON NUMERICAL COGNITION PROPOSED DURING FUNDING PERIOD

Saxe, G. B. The Development of Numeration Concepts: The Effects of Language Background, Mother-Child Interaction, and Meta-Cognitive Processes. Submitted to National Institute of Education.

Saxe, G. B., & Moylan, T. A Developmental Analysis of Indigenous Systems of Quantification Among the Oksapmin. Submitted to Department of Education, Papua New Guinea.

Saxe, G. B. The Origins of Counting in Social Interaction. Submitted to PSE-BHE, City University of New York.

Section I.

Overview Articles

Section II.

Interrelations Between Notational Counting and Number Conservation

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Figurative and Operative Aspects of Number Representation:

A Developmental Analysis

Geoffrey B. Saxe, Warren Cohen and David Rindskopf

The Graduate Center of the City University of New York

Running head: Figurative and Operative Aspects

Figurative and Operative Aspects

Abstract

In this study 4- and 5-year-olds' performance on two number conservation tasks was assessed, one in which children represented the number in a target set using physical tallies and the other in which they represented the number in a target set using verbal numerals. Results support the hypothesis that conservation with verbal numerals is understood prior to conservation with physical tallies. Results also tend to support the hypothesis that the set size of the target influences the ability to conserve number to a greater extent using physical tallies than using verbal numerals. The findings are discussed with respect to the interplay between figurative aspects of number representation and developing operative structures.

Figurative and Operative Aspects of Number Representation:

A Developmental Analysis

Piaget has distinguished between two types of cognitive activities with which a subject constructs knowledge (Piaget, 1970). "Operative" activity refers to the subject's transformation of reality into logical structures (e.g., through classifications or seriations) whereas "figurative" activity refers to the subject's attempt to faithfully reproduce reality in a particular representational medium (e.g., imagery or drawing). In the present study, the distinction between operative and figurative activity is extended to the domain of numerical representation. It will be argued that there is an inherent relation between figurative and operative activity in any act of numerical representation and that figurative differences in the medium in which children represent number (verbal numerals or physical tallies) influences their operative understanding of numerical relations such as number conservation.

The argument that there is an inherent relation between operative and figurative aspects of number representation can be explicated through an example. Consider two subjects who both produce a representation of a target set of elements. The first subject counts the target set and the second subject matches the target one-to-one with a set of physical tallies (e.g., beads). The figurative aspect of each subject's activity is the production of a "faithful copy" of the target array. This activity is figurative in that each subject

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follows elements, one by one, in order to construct an accurate reproduction in a particular representational medium (verbal numerals or physical tallies). The operative aspect of each subject's activity is the summation of the target array into a numerical value (in the form of a verbal numeral or a physical tally), a summation which is conserved if the array is spatially redistributed. This aspect of number representation is operative since the summation is not in the array itself but is an organization imposed on the array by the subject. Thus, in any form of number representation, a subject simultaneously provides an accurate reproduction of the array in the form of a signifier whether it be a verbal numeral or physical tally (figurative aspect) and transforms an array into a summation (operative aspect).

The above example illustrates that figurative and operative activities are inherently related in any act of number representation. The example also illustrates that there can be figurative differences in the media which a subject uses to embody number. For instance, physical tallies entail a spatial embodiment whereas verbal numerals do not. It will be argued below that if children use physical tallies to represent the number in a target set, they will have more difficulty understanding conservation than if they use verbal numerals to represent a target set.

Consider Piaget's classic task format for assessing conservation. Piaget (1952) required the child to establish a correspondence between physical tallies and a target set; one array was then spatially redistributed and the child was faced with two arrays which differed in spatial extent, the tallies and the target. Piaget, as well as many others who have replicated this study,

found that young children have considerable difficulty in resolving the contrast between the marked differences in the spatial extent of the two arrays and their numerical equivalence. This difficulty suggests that if a child were to use verbal numerals instead of physical tallies to represent number, children's confusion of number with spatial extent should be minimized. In the case of verbal numerals, the child would compare the transformed array to a verbal representation of number which he arrived at through counting the target. This verbal representation has no spatial embodiment. Hence, since the child's spatial conflict should be minimized, children should understand conservation in the medium of verbal numerals prior to physical tallies. This is one way in which figurative characteristics of children's number representation should interact with their developing understanding of conservation.

There is, in fact, support for this developmental ordering of conservation across these two media of representation. Greco (1962) reported that when, in the classical conservation task, children are faced with two arrays of elements, one of which has been spatially transformed, some children claim that the number names of the arrays are the same, but one row nonetheless contains more elements (Greco called this conceptual level "quantity"). In other words, when children use a verbal medium to represent the two arrays in the classic conservation task, at least some children evidence a "precocious" understanding of conservation. Inhelder, Sinclair & Bovet (1974) reported a similar phase that they define as transitional between non-conservation and conservation. During this phase, following a non-conservation response in a classic conservation task, some children can tell the interviewer the correct number of

elements in the transformed array after counting their copy. Finally, Elkind (1967) and various follow-up studies (see Brainerd & Hooper, 1975; Miller, 1978 for discussions and reviews) have shown that if only one array is presented and spatially redistributed, some children evidence an understanding of conservation at a younger age than if two arrays are presented and one is transformed (as in the case of physical tallies).

There is another way in which representing media with verbal numerals as opposed to physical tallies may influence children's understanding of conservation. There are theoretical grounds for arguing that the set size of the target array will influence children's understanding of conservation with physical tallies but not with verbal numerals. Piaget (1952) has claimed that the young child has difficulty maintaining a conception of the discrete character of the elements of a set when the child considers the whole set, and that, as a result, children evaluate conservation on the basis of an ordinal index of quantity—spatial extent. An implication of this analysis is that in the case of physical tallies, children will understand conservation for small set sizes prior to large set sizes since small sets provide children with a perceptual support for coordinating the relation between the elements and the whole set. That is, with small sets (e.g., three elements), unlike large sets (e.g., 100), the perception of the whole array provides information about the discrete number of elements. In contrast, with verbal numerals, set size should not influence children's understanding of conservation in this way. Since the numeral does not have a spatial embodiment, the child should be less swayed by changes in spatial extent at any level of set size.

In sum, the hypotheses presented are that children's understanding of conservation is affected by both the medium of number representation and the set size of the target array. Specifically, children should understand conservation with verbal numerals prior to conservation with physical tallies, and conservation judgments should be affected by set size to a greater extent with physical tallies than with verbal numerals. The purpose of the study presented below is to test this formulation.

Method

Subjects

The sample consisted of 93 children (54 4-year-olds and 39 5-year-olds). All children attended middle class nursery and elementary schools in the New York City area.

Procedures

Prior to the administration of the conservation tasks, the interviewer asked the child to count to 10. If the child could not count to 10, the child was not included in the sample.

Children who were selected to participate in the study were assigned randomly to one of three set size conditions: 3, 5, or 8 elements. Each child received four trials on the verbal numerals task and four trials on the physical tallies task at the specified level of set size. The individual trials for each task were interpolated with one another in blocks of two, so that children did not receive the four trials for any one task type consecutively. In addition, the order of administration of the tasks was counter-

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balanced so that half the subjects received two trials of the verbal numerals task first, while the remaining half received two trials of the physical tallies task first.

Conservation task: Verbal Numerals. The interviewer presented the child with an array of either 3, 5, or 8 green beads, .75 cm in diameter which were strung along a wire supported by two uprights 56 cm apart. The initial length of the array was 25 cm. The interviewer said, "I want to know how many beads there are. Help me count them." The interviewer counted the beads with the child and then asked, "So, how many do we have?" If the child did not repeat the number just counted, the interviewer recounted the beads with the child until the correct answer was given. The interviewer then reminded the child how many beads he had counted, "So there are (the set size of this condition) beads." The interviewer then said, "Now watch what I'm going to do," as he spread the beads out so the array extended to 50 cm. The child was then asked, "Now, are there (set size) beads, or are there a different number?" and the child's answer was recorded. If the child answered "a different number" the child was asked, "How many are there now?" If a child attempted to count the beads, the interviewer interrupted the child's counting by temporarily screening the array from the child's view and saying, "Just guess how many there are."

The interviewer then repeated the child's last response saying, "So there are (child's last response even if incorrect) beads. Now watch what I'm going to do," while saying this, the interviewer compressed the array until its length was 15 cm, and then asked "Now are there still (child's last response) beads or are there a different number?"

Conservation task: Physical tallies. The interviewer presented the child with the identical array of green beads supported on the upright which was used in the verbal numerals task. In addition, the child was given a set of orange beads which contained two more beads than the green set. (For each subject, the set size used in the physical tallies and the verbal numerals task was the same.) The array of green beads was initially set at 25 cm. The child was asked to put out one of his orange beads for each of the interviewer's green beads, lining them up in a row directly in front of the green beads. If the child failed to set up a one-to-one correspondence, the interviewer helped the child. Once the two arrays were in one-to-one correspondence, the interviewer asked, "Now, do we both have the same number?" Once the child answered affirmatively, the interviewer then said, "Now watch what I'm going to do," while he was spreading the green array so that its' length was 50 cm. The child was then asked, "Now, do we still have the same number or does one of us have a different number?"

The interviewer then said, "Watch what I'm going to do" as he compressed the array of green beads until its' length was 15 cm. The child was then asked, "Now, do we still have the same number, or does one of us have a different number?"

Scoring

Conservation task: Verbal numerals. Children were assigned a pass or fail score that was determined by whether their numeral judgment after a transformation matched their previous numeral judgment. Since there were four trials, each child received four scores.

Conservation task: Physical tallies. Children were assigned a pass or fail score that corresponded to their conservation judgment on each trial. Since there were four trials, each child received four scores.

Results

Children's responses were used to classify them as either conservers or non-conservers for each of the tasks (physical tallies and verbal numerals). We used two different criteria for assigning subjects to these categories. The conceptually preferred criteria was produced from a statistical procedure which involved a latent structure analysis (cf. Lazarsfeld & Henry, 1968; Goodman, 1974a, 1974b). However, due to some special characteristics of the data which lead to problems in interpreting the latent structure analysis, we used an alternate basis for classifying subjects' response profiles as well. The alternate procedure consisted of defining subjects who produced conservation responses across each trial of a given task as conservers, and those who did not, as non-conservers. Both methods of classifying subjects were then subjected to non-parametric tests in order to test whether both procedures yielded classifications which corresponded with the predicted trends.

The latent structure analyses were designed to tell us if two categories (conservers and non-conservers) were sufficient to describe subject's response profiles, and if so, to classify subjects into one of these two categories. When large sample sizes are used, the goodness-of-fit of such classifications can be tested, and if the twofold classification does not fit, an alternative model (perhaps with an additional class) can be tested. The small sample size in the present study reduces the power of the latent structure analysis, and

probably increases the number of zero cells in the tables. A strategy frequently used to remove cells with no observations, and to reduce bias in the estimates of model parameters and their standard errors is to add a small value to each cell frequency. In our analyses, we added 2.

The results of the latent structure analysis for each of the set sizes for the physical tallies and verbal numeral tasks is presented in Table 1. In order to give some idea about the power (or lack of it) in the tests of the two-class models, Table 1 also contains goodness-of-fit statistics for a test of a one-class model (which is just a test of independence of responses to the items).

Insert Table 1 about here

Table 2 contains the classifications of subject's based upon the latent structure analysis and Table 3 contains the subject classifications based upon the perfect/non-perfect criterion. Sign tests were used to analyze the relationships in each table. For the latent structure classification these tests revealed that verbal numerals was the easier task at set size 5 ($N = 5$, $\bar{x} = 0$, $p < .05$) and set size 8, ($N = 15$, $\bar{x} = 0$, $p < .01$), but that there was no difference between these conditions at set size 3 ($N = 8$, $\bar{x} = 2$, $p < .15$). In contrast for the perfect/non-perfect classification scheme, sign tests revealed that at every level of set size, conservation in verbal numerals was mastered prior to conservation in physical tallies (set size 3, $N = 11$, $\bar{x} = 1$, $p < .01$; set size 5, $N = 6$, $\bar{x} = 0$, $p < .02$; set size 8, $N = 8$, $\bar{x} = 0$, $p < .01$).

Together these analyses indicate that conservation with verbal numerals is understood prior to physical tallies, and that this difference may be more pronounced at large than small set sizes.

Insert Tables 2 and 3 about here

Chi-square tests were used to determine whether set size influenced children's understanding of conservation on each task condition. For the latent structure analysis, this test revealed that on verbal numerals conditions children performed better on set size 3 than set size 5 ($\chi^2_{(1)} = 3.84, p < .05$), but that there were no differences in their performance between the set size 5/8 conditions ($\chi^2_{(1)} = 1.15, p = \text{n.s.}$) nor set size 3/8 conditions ($\chi^2_{(1)} = .39, p = \text{n.s.}$). For the physical tally conditions, these tests revealed that children performed better on the set size 3 than the set size 5 condition ($\chi^2_{(1)} = 4.20, p < .05$) and the set size 8 condition ($\chi^2_{(1)} = 10.91, p < .01$); there was no difference in children's performance between the set size 5/8 conditions ($\chi^2_{(1)} = 1.15, p = \text{n.s.}$). Chi-square tests were also performed for set size on the perfect/non-perfect classification scheme, and the relationships were slightly altered. On the verbal numerals task these tests revealed that children performed better on the set size 3 than the set size 8 condition ($\chi^2_{(1)} = 5.98, p < .02$) although there were no differences in performance between the set size 3/5 ($\chi^2_{(1)} = 2.92, p < .10$) or 5/8 conditions ($\chi^2_{(1)} = .26, p = \text{n.s.}$). On the physical tallies tasks, these tests revealed a similar relationship. Children performed better on the set size 3 than set size 8

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condition ($\chi^2_{(1)} = 4.29, p < .05$), and no differences in performance between the set size 3/5 ($\chi^2_{(1)} = .58, p = \text{n.s.}$) or 5/8 ($\chi^2_{(1)} = 1.15, p = \text{n.s.}$) conditions. In sum, the pattern of relations across these types of classification schemes indicates that increasing set sizes depresses performance within both types of task conditions and that set size may affect performance more strongly in the medium of physical tallies than in the medium of verbal numerals.

Discussion

In the present study we examined whether figurative differences in the way in which number is represented lead to systematic differences in children's ability to structure operative concepts of conservation. We argued that if children represent an array in a verbal medium, they avoid the confusion between spatial extent and number which arises when number is represented using physical tallies. The findings supported this formulation. When children represented the number of elements in a target array with verbal numerals as opposed to physical tallies, they understood concepts of conservation at a younger age. Moreover, we also found, as predicted, that the size of the target set affects the children's ability to understand conservation when physical tallies are used to represent number, but less so when verbal numerals are used.

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A criticism of this study may be that in the verbal numeral task we have not actually assessed children's ability to conserve. It may be that some children initially counted the array but did not attribute to the last numeral in their count any quantitative significance. When, after the array was transformed, these children continued to use the same numeral response to the interviewer's question, the children may only have been using the numeral to refer to some qualitative aspect of the array such as the "same bunch of beads," and not to the numerosity of the set. Thus, one might argue that children who seem to be conserving only in the verbal medium condition may merely be using numerals as a means of expressing an understanding of a qualitative identity and not an understanding of quantitative invariance.

There is evidence from previous studies, however, which indicates that some "pre-conserving" children in the classical conservation task (physical tallies) do use counting effectively in tasks that involve notions of numerosity. Several studies indicate that prior to conserving on the classical number conservation task children use counting to mediate both numerical comparisons of sets and numerical reproductions of a model set (Russac, 1978; Saxe, 1979). Thus, it seems reasonable to suppose that when, in the present study, children judged that the numeral does not change over spatial transformations, they were expressing an understanding that quantity was conserved. This then would be a true conservation in which the child must, to use Piaget's (1967) terms, "compose" quantitative variations (nothing added, nothing taken away = same amount) and not merely a concept of qualitative identity.

The results of this study point to the need for further research in two related directions. First, to date we have little understanding of systematic ways in which children's use of different representational media may facilitate or inhibit their structuration of similar operative concepts. Further understanding of this interplay would provide information about basic processes in the development of numerical cognition. Second, it is likely that children draw upon one medium of representation in which they have achieved an understanding to help structure numerical concepts in another medium. For instance, children who are beginning to learn the arithmetic properties of numerals often employ physical tallies to represent arithmetic transformations. The use of one medium to support concept formation in another medium may be a mechanism which facilitates cognitive developmental change not only in the domain of number representation, but in other domains of cognition as well. A deeper understanding of how children establish functional links between media may therefore be of considerable import not only to our general understanding of cognitive development, but in furthering educational practice.

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Figurative and Operative Aspects

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Table 1

Latent Structure Analyses for Physical Tallies and Verbal Numerals Tasks

Task	Set Size	One Class Model			Two Class Model		
		χ^2	df	p level	χ^2	df	p level
Physical Tallies	3	67.76	11	< .001	4.04	6	.65
Physical Tallies	5	86.47	11	< .001	10.04	6	.10
Physical Tallies	8	57.53	11	< .001	8.96	6	.20
Verbal Numerals	3	54.72	11	< .001	2.07	6	.95
Verbal Numerals	5	93.40	11	< .001	6.13	6	.50
Verbal Numerals	8	48.08	11	< .001	6.26	6	.50

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Table 2

Response Profiles on Verbal Numerals and Physical Tallies Tasks

According to Latent Class Analysis

Set Size	Verbal Numerals+	Verbal Numerals+	Verbal Numerals-	Verbal Numerals-
	Physical Tallies+	Physical Tallies-	Physical Tallies+	Physical Tallies-
3	20	6	2	3
5	13	5	0	13
8	8	15	0	8

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Table 3

Response Profiles on Verbal Numerals and Physical Tallies Tasks

According to Perfect/Nonperfect Classification Scheme

Set Size	Verbal Numerals+	Verbal Numerals+	Verbal Numerals-	Verbal Numerals-
	Physical Tallies+	Physical Tallies-	Physical Tallies+	Physical Tallies-
3	16	10	1	4
5	13	6	0	12
8	8	8	0	15

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Appreciation is extended to children, teachers and staffs of Crestwood Country Day School, Melville, New York and Hills Elementary School, Dix Hills, New York. Appreciation is also extended to Maryl Gearhart and Alice Klein for useful comments on earlier versions of this manuscript. The research was supported by a grant from the National Institute of Education, G-78-0076. Requests for reprints should be addressed to Geoffrey B. Saxe, Department of Educational Psychology, The Graduate Center of the City University of New York, 33 West 42nd Street, New York, New York 10036.

Empirical Operations and Logical Necessities:
The Role of Counting in Developing Conservation Concepts

Geoffrey B. Saxe and Warren Cohen

The Graduate Center of the City University of New York

Running head: Empirical Operations

Abstract

This study examines how 3-, 4-, and 5-year-old pre-conserving children resolve the tension between their belief in non-conservation and the information which they produce by counting sets before and after spatial transformation. The results indicated that 5-year-olds, but not 3- and 4-year-olds tended to change their non-conservation judgments to conservation judgments after counting sets and that 3- and 4-year-olds, who miscounted sets, frequently appeared to "purposefully" miscount in order to render their counting consistent with their belief in non-conservation. The significance of the findings for theories about the relationship between empirical counting and logical concepts of conservation is discussed.

Empirical Operations and Logical Necessities:

The Role of Counting in Developing Conservation Concepts

There is a growing body of literature on causal relations between the child's use of counting to represent number and children's understanding of number conservation. A fundamental issue is whether children can arrive at logical concepts of number conservation through the empirical information they gain by counting sets of objects. Some researchers have argued that counting is a necessary and critical condition for number conservation (e.g., Klahr & Wallace, 1976); on this view children "abstract" logical concepts of conservation by counting arrays before and after spatial transformations. Others have argued for a weaker relation, pointing out that empirical information which is necessarily subject to errors cannot be a direct source of logical concepts which are true by virtue of their logical necessity (e.g., Saxe, 1979). The present study was conducted in order to determine how pre-conserving children integrate information produced by counting sets before and after spatial transformations with their developing understanding of conservation. It was expected that the way in which pre-conserving children handle the conflict between information produced through their counting and their belief that number is not conserved should serve to inform existing models of the relationship between counting and conservation.

MethodSubjects

The sample consisted of 55 children who attended middle class nursery and elementary schools in the New York City area. There were 22 3-year-olds

(mean age = 3 years, 6 months, standard deviation = 3 months), 19 4-year-olds (mean age = 4 years, 5 months, standard deviation = 3.1 months), and 14 5-year-olds (mean age = 5 years, 6 months, standard deviation = 4.6 months).

Procedure

Assessment phase. The interviewer presents the child with an array of either 5 or 8 beads, .75 cm in diameter, which were strung along a wire supported by 2 uprights. The interviewer arranged the beads so that they extended a distance of 10 cm from the left hand side of the upright. The interviewer then said, "I want to know how many beads there are. Help me count them." The interviewer and the child then counted the beads together. To insure that the child had been attending to the count, the interviewer asked, "So how many do we have?" and corrected the child if necessary. The interviewer then redistributed the elements on the wire four consecutive times (two expansions and two compressions), and after each consecutive redistribution, asked the child the number of elements. Only those children who judged that there were a different number of beads on at least 3 of the 4 redistributions were included in the study.

Conflict phase. Using the same apparatus as in the assessment phase, the interviewer again arranged the beads so that they extended a distance of 10 cm from the left hand side of the upright. As in the previous phase, the interviewer then said to the child, "I want to know how many beads there are. Help me count them." The interviewer and the child then counted the beads together. To insure that the child had been attending to the count, the interviewer asked, "So how many do we have?" and corrected the child if necessary.

To emphasize the number of beads just counted, the interviewer said, "So we have (number of beads)." The interviewer then said, "Now watch what I'm going to do," and spread the beads out so that they extended for a distance of 20 cm. The interviewer then asked the child, "How many are there now?" and recorded the child's answer. The child was not allowed to count. If the child refused to give an answer, the interviewer encouraged the child by saying, "Just guess." If the child attempted to count the beads, the interviewer interrupted the child's counting by temporarily screening the array from the child's view saying, "Just guess how many there are." Once the child offered a judgment, the interviewer asked the child to recount the beads. If children miscounted, their miscount was recorded and the interviewer helped the child produce an accurate count.

The interviewer repeated the above procedure in spreading the beads to 30 cm and then to 50 cm. The final transformation to 50 cm was not followed by a count.

Results and Discussion

No differences were found between set size conditions in either subjects' counting accuracy or their tendency to change their belief from non-conservation to conservation. Consequently, children's performances on the two set size levels were pooled.

There was a moderate degree of inaccurate counting among the children, and the frequency of miscounting was related to age group ($\chi^2_{(2)} = 9.97, p < .05$). Of the 22 3-year-olds, 15 children counted inaccurately on at least 1 trial (68%). Of the 19 4-year-olds, 9 children counted inaccurately on at least 1 trial (47%), and of the 14 5-year-olds, 2 children counted inaccurately on at least 1 trial (14%). Of particular interest was that some of the 3- and 4-

year-olds who counted inaccurately appeared to "purposefully" miscount. That is, their empirical count of the array matched their previous inaccurate prediction about the value of the array. Of the 3-year-olds who miscounted, 60% "purposefully" miscounted. Of the 4-year-olds who miscounted, 67% "purposefully" miscounted. Neither of the two 5-year-olds who miscounted "purposefully" miscounted.

As noted earlier, whenever a subject miscounted, the interviewer helped the child count the set accurately. Thus, all children were provided with an accurate count of the set. Children were categorized as one of three types based upon their predictions following each of the two feedback trials: (1) those who did not change their non-conservation predictions at all; (2) those who changed to predictions of conservation after the second of the two feedback trials; and (3) those who changed to predictions of conservation after both feedback trials.¹ Table 1 contains the subject classifications at each age level. Chi-square analyses indicated that children in different age groups differed on whether they changed their predictions to conservation (pooling categories 2 and 3, $\chi^2_{(2)} = 12.05$, $p < .01$). Follow-up analyses indicated that there were no differences in the extent to which 3- and 4-year-olds changed to predictions of conservation ($\chi^2_{(1)} = .98$, $p = \text{n.s.}$). In contrast, the 5-year-olds changed to predictions of conservation to a greater degree than the 3-year-olds ($\chi^2_{(1)} = 9.80$, $p < .01$) and the 4-year-olds ($\chi^2_{(1)} = 4.98$, $p < .05$).

Insert Table 1 about here

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The findings of the present study indicate that there is a difference between age levels in how non-conserving children resolve the discrepancy between their empirical counting and their belief in non-conservation. Young children tend to persist in their belief in non-conservation, despite the contrary information which they generate in their counting. They even appear to error in the direction of their prediction by purposefully miscounting in order to support their belief in non-conservation. Older children tend to alter their belief in conservation to accord with their empirical counting. The findings have direct implications for models concerning the role of counting in children's formation of concepts of conservation.

On a "theory of abstraction," children abstract the concept of numeral invariance from information generated through counting arrays before and after spatial transformation. Hence, young children, who frequently count inaccurately, believe that the numerical value of an array varies over spatial transformations. Older children, who typically count accurately, abstract conservation through their empirical counting.

A serious problem with the abstractionist formulation (or any such formulation that posits empirical operations as the direct source of logical concepts) is that it neglects the subject's role in constructing a coherent interpretation of empirical counts. Within the abstractionist framework, there are two possible ways that children could regard the products of their counting, but neither could lead to an understanding of conservation. Children could assume that they always produce an accurate count before and after spatial transformations, since even older children do miscount on occasion. On the other hand,

children could assume that sometimes they err in their empirical counting. If this were the case, however, children would not know when they had or had not produced an accurate count and consequently would not be able to determine whether conservation always occurs following a spatial transformation.

The fact that some of the children appeared to purposefully miscount in the present study even further complicates the picture within the abstractionist framework. If even some children purposefully miscount it is difficult to conceive of how, in general, children come to understand the logical necessity of conservation based solely upon their counting.

An alternative interpretation that more adequately accounts for the findings of the present study is based upon a constructivist theory (see Cassirier, 1953). On this formulation, children do not merely abstract logical concepts of invariance from the products of empirical counts. Rather, children subject the products of their empirical counting to an interpretation, and, this "interpretation" is inherently related to children's "theoretic" understanding of conservation (e.g., whether children believe in the "necessity" of non-conservation or the possibility of conservation). Children's purposeful miscounting, then, is an early manifestation of their attempt to maintain coherence between their theoretic knowledge and their empirical operations, and, developmental changes in children's use of counting to inform their invariance judgments is guided by their changing "theoretic" understanding about the possibility of conservation. In contrast to a "theory of abstraction", then, on the

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constructivist view children's interpretations of their empirical counting becomes the "data" for their developing logical concepts of conservation, rather than their empirical counting itself.

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Footnote

¹Only one subject changed to a prediction of conservation on the first but not the second feedback trial. This subject was not included in the analysis.

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Table 1
 Childrens' Conservation Prediction Profiles Following
 Feedback Trials at Three Age Levels

Age Level	Prediction Profile*		
	--	+-	++
3	17	3	2
4	12	1	6
5	3	4	6

*The symbols refer to the following prediction profiles.

--: Non-conservation prediction after each feedback trial.

+-: Conservation prediction after second, but not first
 feedback trial.

++: Conservation prediction after both feedback trials.

Body Parts as Numerals:
A Developmental Analysis of Numeration
Among the Oksapmin in Papua New Guinea
Geoffrey B. Saxe
The Graduate Center of the City University of New York

Running head: Body Parts as Numerals

Child Development, in press

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Body Parts as Numerals

Abstract

The purpose of this paper is to present an analysis of the acquisition of an indigenous body part numerational system by children who live among remote Oksapmin village populations in Papua New Guinea. The findings reported in Studies 1 and 2 indicate that Oksapmin children progress from pre-mediational to mediational phases in their use of body parts to compare and reproduce number, and that this change generally occurs prior to the development of concepts of number conservation. This trend parallels findings in the United States. The findings reported in Study 3 show that this general change is manifested in culturally specific ways. For instance, Oksapmin children progress from a belief that the numerical relations between any two body parts is determined by their physical similarity (e.g., symmetrical body parts imply numerical equivalence) to the understanding that the numerical relation between any two body parts is determined by their ordinal positions in an enumeration.

Body Parts as Numerals: A Developmental Analysis of Numeration

Among the Oksapmin in Papua New Guinea

This paper presents research on the acquisition of a body part counting system by children who live among remote Oksapmin village populations in Papua New Guinea. The research is motivated by two related concerns. One is to analyze developmental changes in Oksapmin children's use and understanding of their counting system as a notational symbol system for number. A second concern is to analyze the developmental relation between Oksapmin children's use of their counting system and their understanding of number conservation.

The activity of counting involves establishing a one-to-one correspondence between two sets of elements. One set, the "standard," is a conventionally defined list of symbols for number which is always used in counting (e.g., "one," "two," "three,"...). The second set, the "variable," is a target array of elements which varies from one count to another. The subject's aim in counting is to establish a one-to-one correspondence between the two sets such that the standard serves to notate a progressive summation of correspondences with the variable set.

There are some necessary characteristics of all counting systems that allow a subject to notate a progressive summation of correspondence relations. In particular, each numeral of a standard set must be differentiable from all of the others, and each must have a prescribed order of use. Within

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these general constraints, the particular characteristics of counting systems vary from one culture to another. For instance, the Western system consists of an arbitrary set of verbal and graphic numerals with a base-10 structure. The base structure enables the user to generate a very large number of unique and ordered numerals that have a conventional significance. Other cultural groups make use of parts of the body for numerals and the base structures of body part counting systems are known to vary widely (cf. Lancy, 1978; Menninger, 1970; Zaslavsky, 1973). Some systems, such as the Oksapmin's, have no base structure at all.

In the Oksapmin body counting system, twenty-seven body parts are named or enumerated between the thumb on the ^{one} ~~right~~ hand and the little finger on the ^{other} ~~left~~ hand. As the individual ascends up the ^{one} ~~right~~ side of the body in a count, body parts are named, and as the individual descends down the ^{other} ~~left~~ side, symmetrical body parts are named but a prefix is inserted before each body part name. If a greater numeral is required, the individual continues in a loop-like fashion to the wrist of the ^{second} ~~left~~ hand and ascends back up the ^{that} ~~left~~ side of the body (Figure 1). Although individuals often point to specific body parts when counting, it is not necessary to do so. Individuals use this counting system in everyday activities and transactions when a record of the number of a set of elements is needed.

Insert Figure 1 about here

The focus of the present set of studies is on developmental changes in Oksapmin children's acquisition of their counting system, and some of the similarities and differences between the Oksapmin and Western children's acquisition of counting. Various studies conducted in the West indicate that young children use counting simply to enumerate, not to determine or compare cardinal values of sets. For instance, Schaeffer, Eggleston, and Scott (1975) and Gelman and Gallistel (1978) have found that young children who use their standard set to iterate variable sets, often do not treat the last numeral recited as a representation of cardinal value. Similarly, Saxe (1977, 1979) found that young children may use the standard set to iterate elements, but nonetheless, use "pre-mediational" forms of counting when required to compare or reproduce sets. That is, young children may count arrays in the context of numerical comparisons or reproductions while basing their numerical comparisons or reproductions on non-numerical properties of collections, such as spatial extent.

The developmental relation between the child's acquisition of notational counting and number conservation has been a source of some controversy. Piaget argued that in order for children's counting to achieve the status of a notational symbol system children must understand number conservation; that is, children must understand the number of elements in a set does not change if the elements are redistributed (spread apart or crowded together). Piaget's argument rested on his assumption that in order for numerals to serve a numerical function, numerals must refer to correspondence relations which are conserved over spatial transformations. But there is a problem with this argument in that numerals can be used solely to represent correspondence relations with static arrays of elements. For instance, if an individual counts two sets of items in order to compare their numerical values, the two cardinal values would be used to represent the two static sets. In neither

case would the individual need to consider whether the correspondence relations between numerals and elements were altered over a spatial transformation of the arrays. It is plausible, then, that at least some children who do not understand number conservation use their counting as a notational system to compare and reproduce static arrays. Various researchers have, in fact, produced findings consistent with this latter formulation. For instance, both Russac (1978) and Saxe (1979) have demonstrated that children use counting as a notational symbol system to compare or reproduce static sets prior to understanding number conservation.

The findings from the West suggest three general hypotheses about Oksapmin children's development of counting and its relation to their understanding of number conservation. First, Oksapmin children may experience a similar pre-mediational phase in their acquisition of counting as a means to compare and reproduce sets. Second, this phase may be manifested in culturally specific ways. For instance, since the body part system has a spatial representation, during a pre-mediational phase Oksapmin children may confuse physical similarity of body parts with numerical equivalence. Third, as in the West, the Oksapmin children's shift to mediational counting strategies to represent static sets may occur prior to their understanding of number conservation.

The concern of the first two studies described below was to determine whether Oksapmin children progress from pre-mediational to mediational uses of their body part counting system to compare and reproduce sets, and whether this development occurs prior to the development of concepts of number conservation. The concern of the third study was to determine whether a change from pre-mediational to mediational forms of counting would be manifested in children's confusions that are specific to the acquisition of the Oksapmin system, and the relation of these specific confusions to children's understanding of number conservation.

STUDY 1

MethodPreliminary Procedure

In order to adapt procedures used in the United States to village life, a series of preliminary phases were completed. First, a week was spent in an Oksapmin village becoming acquainted with the people and, consulting with two in-resident anthropologists. Second, tasks were developed that were relevant to village life and, with an assistant, translated from English to Oksapmin. A back translation procedure was used to accomplish the translation. In this procedure, a sentence, or a series of sentences, was stated in English and the assistant translated the sentences into Oksapmin. The assistant's translation was tape recorded. After a series of task translations were completed, the assistant was required to back translate the tape recording into English. When necessary, the translations were altered until the procedure yielded the reliable back translations from Oksapmin. Third, Oksapmin children at different age levels were interviewed to see in what ways the procedures could be improved. Fourth, the back translation process was repeated using the modified procedures. These pilot phases led to the procedures described in the three studies below.

Subjects

Oksapmin communities are small remote village populations located in the highlands of the West Sepik Province of Papua New Guinea. There are no roads to the area and the only means of access is by single engine aircraft and

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travel by foot. For subsistence, individuals raise pigs, cultivate a variety of vegetable crops (e.g., varieties of sweet potatoes and taro), and hunt for small game using bow and arrow. Many of the people still wear traditional dress (penis sheath or grass skirt) and use traditional methods of subsistence. Social change due to Western influence is increasingly apparent. There are now mission stations scattered in the region and many villages have a Christian chapel with a native priest. There is minimal use of currency. When currency is used, it is generally to purchase goods at tiny trade posts.

The sample included 28 children who were classified in one of three age groups; these included 13 5- to 9-year-olds, 6 10-year-olds, and 9 11- to 16-year olds. Although some of the children's ages were determined through missionary birth records, others were estimates based upon visual comparisons of children of known ages with children of unknown ages. Since many of these ages were only approximations, a procedure described later in the paper was used that would allow for additional confidence in statistical comparisons between age groups. Children who participated had a maximum of four months of schooling.

Materials

All materials used in the tasks were indigenous to the area.

Counting tasks. Two types of counting tasks were used, a comparison task and a reproduction task. (a) Comparison task: The materials for the comparison task consisted of small stones (approximately 2 cc and small pieces of bamboo (about 1 cm in diameter and 2 cm long). (b) Reproduction task:

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The materials for the reproduction task consisted of small pieces of clay about 1 cm in diameter) and small stones (about 2 cc).

Number conservation tasks. Materials for the number conservation tasks were the same as those used for the comparison task.

Procedures

All tasks were administered via the assistant in the Oksapmin language. The order of administration of the counting and conservation tasks were counter-balanced; however, for each task, children received three levels of set size conditions in a constant order: small (3-4 elements), intermediate (6-7 elements), and large (8-9 elements). It was assumed that this order would permit maximum success. Although an attempt was made to administer each subject all of the conditions, for various reasons this was not possible (e.g., some of the younger subjects were too shy to complete the test battery).

Counting tasks. (a) Comparison: In each of the three conditions, children were shown parallel rows of stones and sticks with the longer row containing the fewer number of elements. Children were asked whether there was the same number of sticks and stones or whether there was a bigger number of stones or a bigger number of sticks. If children did not count spontaneously, their judgment was recorded and they were asked probe questions to encourage counting (e.g., count and show). (b) Remote placement: Children were required to reproduce 3, 6, and 9 element collections. In each case, a set of elements was placed behind children and they were told that they should put out just the same number of elements on the floor in front of them. If children did not count spontaneously to solve the task, we suggested counting to them (e.g., count and show).

Analyses of children's counting behavior on these tasks (with the exception of the small set size condition) paralleled analyses completed on Western children's acquisition of counting (Saxe, 1977, 1979). Children's counting strategies were categorized as either pre-mediational or mediational. Pre-mediational counting strategies were defined as those in which children counted on a task, but did not base their numerical comparisons or reproductions on their counting. For instance, children might count only a single array and offer a comparative judgment, or count two arrays continuously as if they were one and offer a comparative judgment. Mediational counting strategies were defined as those in which children based their numerical comparisons or reproductions upon their counting. For instance, a child might count both arrays and judge that the array to which they assigned the greater numeral contained the greater number of elements.

For the small set size conditions, children's performances were evaluated for the correctness of their result rather than whether they used a "pre-mediational" or "mediational" counting strategy. The reason for this was that verbal probes often elicited a "pre-mediational" counting strategy across age groups on the small but not the large set size conditions. It was suspected that children who produced an accurate evaluation on the small set size conditions, regardless of their overt counting strategy, achieved a covert solution to the task. Therefore, in order to avoid misclassifications of subjects, only the accuracy of judgment was used as a criteria of maturity for the small set size tasks.

Number conservation task. As in the case of the counting tasks, the number conservation task was also administered at three levels of set size (3, 6, and 9 elements). Children were presented with a linear array of sticks and asked to put out just the same number of stones in front of the sticks. If children did not produce a one-to-one correspondence, their reproductions were recorded, and a one-to-one correspondence was produced for them. Once children confirmed that the two rows contained the same number of elements, the row of sticks was spread apart and children were asked whether there was the same number of sticks and stones or was there a different number of sticks and stones. In every case, children's judgments were probed for an explanation, but appropriate explanations were not forthcoming in virtually all of the interviews. In general, whether children offered a conserving response or not, they justified their judgment with the response, "That's what I see," or "I know," or "I don't know." Since children did not offer adequate verbal explanations, a non-verbal check of their understanding of their judgments was performed after three-quarters of the subjects had been interviewed. Following a conservation judgment, children were told to "watch what I am going to do," whereupon the interviewer subtracted a middle element from the longer array. Children were then asked whether both rows contained the same number or whether there were more sticks or more stones in one of the rows. This procedure revealed that a couple of children who offered conservation judgments, also judged that subtraction did not change the value of the array. Since these subjects may have been misclassified

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as conservers, they were not included in the tabulation of the results. In the subsequent studies, a more stringent criteria was used to classify children's conservation responses.

Results

In the statistical analysis for the age trends, two age groups are compared, children 9 years and younger and children 11 years and older. Children who were classified as 10-year-olds were excluded from these age group comparisons because of lack of confidence in the children's exact ages. However, when comparisons are made across tasks (for instance, comparing children's performances on counting and their performances on conservation tasks) the 10-year-olds are included in the analysis since age is not used as an independent variable on these comparisons.

Counting Tasks

The data presented in Table 1 contain the distribution of children's performance on the small set size numerical comparisons and reproductions conditions. Fisher exact tests revealed a significant effect for age on the comparison task, $p = .02$, and that the age trend approached significance on the reproduction task, $p = .06$.

Insert Table 1 about here

Table 2 contains the frequency distribution of children's use of pre-mediational and mediational counting strategies on the intermediate and large set size conditions of the comparison and reproduction tasks. As

expected, young Oksapmin children tended not to use their counting to mediate their solutions to these tasks. The older age group did use counting to mediate the performances. Fisher exact test revealed that these age trends were significant for each of the conditions (reproduction task set size 6, $p = .01$, set size 9, $p = .02$; comparison tasks, set size 6/7, $p = .001$, set size 8/9, $p = .001$). A comparison of the data in Tables 1 and 2 indicates that greater proportions of children produced mature solutions for the small set size tasks; however, these comparisons did not approach statistical significance using a Sign test (Sigel, 1956).

Insert Table 2 about here

Relations Between Performance on Counting and Conservation Tasks

Children's conservation responses were categorized as one of three levels. These levels are consistent with Piaget's (1952) scoring criteria. No conservation responses were defined as a failure to establish a one-to-one correspondence between the model array and the available set of elements. Transitional responses were defined as establishing a one-to-one correspondence between the model array and the copy, but the judgment that the number changed following the spatial transformation. Conservation responses were defined as a judgment that the numerical relation between the two arrays did not change following the spatial transformation.

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Tables 3 and 4 contain a cross-tabulation of children's performances on the notational counting and number conservation tasks at intermediate and large set sizes. An analysis of children's profiles across the notational counting and conservation tasks indicate that a larger number of children used mediational counting strategies and did not understand number conservation than who did not use mediational counting strategies and did understand number conservation. Sign tests indicated that this trend only approached significance on both the comparison and reproduction tasks at intermediate set sizes (reproduction task: $N=7$, $x=1$, $p < .07$, comparison task: $N=7$, $x=2$, $p < .23$), but at large set sizes this trend achieved statistical significance (reproduction task: $N = 9$, $x = 0$, $p < .002$ comparison task: $N = 6$, $x = 0$, $p < .02$). Although these data do not provide strong support for the claim that there is an invariant sequence between the acquisition of counting and number conservation since several counterexamples were found, the findings do support the claim that Oksapmin children can use counting to mediate comparisons and reproductions of number prior to understanding number conservation.

Insert Tables 3 and 4 about here

It is important to note that two children used pre-mediational counting strategies and did not successfully answer the check questions for "pseudo-conservation"; these children are not represented in the tables. Since the "pseudo-conservation" check was not used with all (or even a majority) of the children, and since the counting measures often required probing to

elicit counting strategies, a second study was conducted using a new population of children to determine whether the findings for the large set size conditions could be replicated.

STUDY 2

In this study, the approximate ages of the children interviewed were older than in Study 1, and the procedures that were used were improved based upon the problems encountered in the administration of the procedures described in Study 1. Only the large set size conditions were used in this study.

Method

Subjects

Twenty seven children were interviewed. Their ages were estimated in the same manner as in Study 1, and ranged between approximately 8 and 16 years. As in Study 1, none of the children had received more than four months of schooling.

Materials

The materials used were identical to those used in the counting and conservation tasks described in Study 1, with the exception that on the comparison task, the clay balls were used instead of the sticks and stones.

Procedures

The order of administration of the following counting and number conservation tasks was counterbalanced.

Counting tasks. (a) Comparison task: The comparison task was altered in the following manner in order to reduce the need for probe questions to elicit counting. Two rows of clay balls were presented about 5 meters apart

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from one another. Each row was about 2.5 meters from the child. One of the rows was about 45 cm in length and contained 9 elements. The second row was about 13 cm in length and contained eleven elements. Both rows were screened from the children's view such that they had to walk to the array in order to see each set of elements. Children were told that there were two brothers who both had gardens of sweet potatoes. One lived in a village over here (the interviewer points to the screened array on one side of the child) and had those sweet potatoes (the child is asked to get up and look at one group of clay balls), and the other lived in a village over there, and had that many sweet potatoes (the child is asked to look at the other group of clay balls and then to return to his initial position). The interviewer then asked the child to count to find out if the two brothers had the same number of sweet potatoes or if they had different numbers. The child's counting strategy and counting accuracy were recorded as in Study 1. (b) Reproduction task: The reproduction task was administered in a similar manner to that described in Study 1 with two exceptions. First, the model array was placed behind a screen 10 feet behind the child, and, in the initial instructions, the child was told to use counting to help him make the numerically equivalent copy (e.g., "Count the stones over there and put out the same number of these sticks over here.") Again, these revised procedures reduced the necessity of eliciting a counting strategy from children. Children's counting strategies were categorized with the same procedure used in Study 1.

Conservation task. The conservation task was revised in several ways. First, two spatial transformations were used in the assessment, one expansion

and one compression. After each spatial transformation, the child was questioned about conservation as in Study 1. If a child offered a conservation judgment, the child was told to "watch what I am going to do." The interviewer then removed one element from an array and the child was asked whether there was still the same number or whether one had more. Children only received the large set size condition.

Children's responses were categorized as one of three types. Conservation responses were defined as those in which the child offered a conservation judgment across each of the two transformations. Transitional responses were defined as those in which the child responded correctly to only one of the two transformation questions. No conservation responses were defined as not satisfying the criteria for inclusion in the first two categories. Only one child who offered conservation responses across both trials did not respond to the check questions adequately; this child was not included in the data analysis.

Results and Discussion

Table 5 contains the crosstabulation of children's notational counting and conservation levels. As shown in the table, of the children who understood conservation, all used mediational counting strategies. In contrast, of the children who used mediational counting strategies, only a portion of them understood number conservation. The distribution of responses supports the trend found in Study 1 which indicated that children tend to develop mediational counting strategies to compare and reproduce number before they achieved an understanding of number conservation (sign tests, combining the "transitional" and the "conservation" responses, comparison task/conservation, $N = 17$, $x = 0$, $p < .001$, reproduction task/conservation, $N = 16$, $x = 0$, $p < .001$).

Insert Table 5 about here

In Studies 1 and 2, two types of developmental sequences previously found in the Western child's acquisition of counting were replicated in Oksapmin children's acquisition of their body part numerational system. First, the findings from Study 1 indicated that Oksapmin children, as do their Western counterparts, progress from the use of pre-mediational to mediational counting strategies in the acquisition of their indigenous numerational system. Young Oksapmin children who knew the elements of their standard set, nonetheless, did not use the elements to mediate numerical comparisons and reproductions. Older Oksapmin children, in contrast, did use body parts to serve this numerical function. However, unlike middle class children from the United States who use counting to mediate comparison and reproductions of number by 5 or 6 years of age (cf. Russac, 1978; Saxe, 1977, 1979), Oksapmin children do not use counting to serve such functions until late childhood.

The second developmental sequence, that between mediational counting and number conservation, was also replicated. The findings from both Studies 1 and 2 indicate that Oksapmin children generally use counting as a notational system prior to achieving an understanding of number conservation. These findings support the general formulation that notational counting may develop prior to conservation since it is used to represent static one-to-one correspondence relations whereas number conservation, in addition to an understanding of static correspondences, requires an understanding that static correspondence relations are unaffected by spatial transformations.

STUDY 3

While the previous studies concerned similarities in Oksapmin and Western children's acquisition of counting, the purpose of the next study was to reveal some of the differences in the acquisition process across the two groups. One salient difference between the Oksapmin and Western systems is that Oksapmin use physical elements for numerical representation, whereas the Western is strictly a verbal system of elements. It was reasoned that if young Oksapmin children do not use their standard set to represent progressive summation of individuals, then they should tend not to evaluate the relative value of body parts on the basis of their numerical equivalence or non-equivalence. Rather, judgments about equivalence may be assimilated to judgments about the physical similarity between body parts. To test this formulation, task procedures were devised in which children compared the numerical values of both symmetrical body parts (e.g., the child's elbows) and asymmetrical body parts (e.g., the child's left elbow and right ear), as well as considered the numerical identity of a single body part reached from different points of origin (e.g., the child's left ear reached from the right thumb as opposed to the left thumb). It was predicted that due to a tendency to evaluate body parts on the basis of their physical similarity, young children would identify symmetrical body parts as signifiers for the same numerical value and that they would also tend to evaluate a single body part arrived at from two different points of origin as a signifier for a single numerical

value. With increasing age, children should tend to evaluate body parts on the basis of their ordinal position in enumerations. Children's understanding of number conservation was also assessed in this study in order to gather additional data on counting/number conservation relations.

Method

The construction of the task procedures described below was accomplished with the aid of a translator, and, as in the previous studies, a back translation procedure was used to help insure the accuracy of the translation. The tasks were pilot tested and then modified prior to taking their final form.

Subjects

Fifty-two children were interviewed who ranged in age from about 7 to 16 years, and were members of the same community as those described in the previous studies, although different children were interviewed in this study than the previous ones. As in the first two studies, ages were determined by missionary records and by comparing children of known age with children of unknown age. Due to the probable error in some of these age estimates, when age group comparisons are made, children who were classified in the middle age group (10-11 years) are omitted from statistical analyses, and only the 7-9 and 12-16 age groups are compared. However, when children's profiles are evaluated across asymmetrical and symmetrical body part task questions, or across the counting and conservation tasks, all subjects are included in the analyses since these comparisons would not be contaminated by categorizing subjects in an inappropriate age group.

Procedures

The counting and conservation tasks described below were administered in a counterbalanced order.

Counting task: asymmetrical/symmetrical body part comparisons. Children were asked to demonstrate how to count and were excluded from the study if they did not enumerate body parts in the conventionally defined sequence. However, if children began counting from the left side of the body, they were told that that was not correct, and that people start from the right side of their bodies. These children were then asked to demonstrate again how to count.

Children who passed the counting task were then told a story about a man who was counting sweet potatoes in his garden. Children were told that one day a man counted to this place on his body (the interviewer and then the translator made a sweeping gesture from children's right thumb to a specified body part on the right side of their bodies). The children were then told that another day the man was counting sweet potatoes and counted to here (the interviewer and then the translator made another sweeping gesture from the children's right thumb to a specified body part on the left side of the children's bodies). Children were then asked if the man counted to the same amount of sweet potatoes (pointing to the two body parts) or different amounts. If children responded different amounts, they were asked which was the bigger amount. Six pairs of body parts were indicated, three symmetrical pairs and three asymmetrical pairs. They were (a) right wrist(6)/left ear(16), (b) right ear(12)/left ear(16), (c) right elbow(8)/left wrist(22),

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(d) right shoulder(10)/left shoulder(18), (e) right side of neck(11)/left biceps(19), (f) right biceps(9)/left biceps(19).

Children's responses were considered correct only if they responded that the man had counted different numbers of sweet potatoes for each question and correctly identified the bigger number. The scores were allied separately for the three symmetrical and the three asymmetrical body part questions.

Counting task: Varying the point of origin. Children were told that although people in their village count from this side (the interviewer points to the child's right thumb) to this side of the body (the interviewer makes a circling gesture around the upper periphery of the child's body to the left hand), in a village over the mountains, people count from this side of the body (the interviewer points to the child's left thumb) to this side of the body (the interviewer makes a circling gesture around the upper periphery of the child's body to the right hand). Children were then asked to demonstrate the way in which people count in their village and the way in which people count in the other village over the mountains. If children confused these instructions, the story was repeated and their comprehension of the instructions was reassessed. Once children passed the comprehension check, they were told the following story. "One day a man from ____ (name of child's village) was counting sweet potatoes in his garden and counted to this place (interviewer gestures from the right side of the child's body to one of three specified body parts). The man from the village over the mountains was also counting sweet potatoes in his garden and counted to this place (the interviewer gestures from the left side of the child's body, to the same one of

the three body parts specified above). The child was then asked whether the two men counted to the same number of sweet potatoes or whether they counted to a different number of sweet potatoes. If children claimed that they counted to different numbers, they were asked who counted to the bigger number.

The three body parts that children were questioned about were the wrist, shoulder and nose. In order for children to achieve a correct score on the wrist or shoulder questions, they had to respond that the two individual's counted to different numbers and to correctly identify the larger numerical value. The question about the nose was used as an additional comprehension check. Children who responded that the nose represented different numbers were excluded from the analysis. (Only two subject's of the total population responded that the nose represented different numbers).

Conservation task: The procedure described in Study 2 was used.

Results and Discussion

Table 6 contains a frequency distribution of children's levels of responses for each age group on the asymmetrical and symmetrical body part questions. Comparisons of the 7 to 9 and 12 to 16 year old age groups revealed that there were significant age differences in performance both on the asymmetrical body part question (Kolmogorov-Smirnov two sample test, $X^2_{(2)} = 11.21$, $p < .01$), and on the symmetrical body part questions (Kolmogorov-Smirnov two sample test, $X^2 = 6.85$, $p < .05$). An analysis of childrens' profiles across the asymmetrical and symmetrical questions also yielded the hypothesized differences.

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Children consistently achieved more correct responses on the asymmetrical as opposed to the symmetrical body part comparisons (sign test $N = 25$, $x = 0$, $p < .001$).

Insert Table 6 about here

Table 7 contains a frequency distribution of the number of correct responses at the two age levels on the point of origin task. Consistent with the responses on the previous task, young children tended to use physical similarity of body parts as a relevant criterion for numerical evaluation, and with increasing age, this criterion shifted to the ordinal position of a body part in an enumeration (Kolmogorov-Smirnov two sample test, $X^2_{(2)} = 7.90$, $p < .02$).

Insert Table 7 about here

An analysis of children's profiles across the counting and conservation tasks revealed that there was some tendency to achieve a mature performance on the counting tasks before the number conservation task. Children more frequently responded correctly on the asymmetrical body part questions prior to number conservation (sign test, $n = 21$, $x = 2$, $p < .001$), although a similar comparison only approached statistical significance for the symmetrical body part questions ($n = 20$, $x = 7$, $p < .14$). Similar comparisons revealed only a tendency for children to respond correctly to the point of origin questions more frequently than they responded correctly to the number conservation questions (sign test, $n = 19$, $x = 6$, $p < .09$).

The findings presented in Study 3 are consistent with those reported in Studies 1 and 2 in that they indicate that there is a general transformation from a pre-mediational to a mediational use of the body part counting system. Furthermore, the findings in Study 3 indicate that this general transformation is manifested in a culturally specific developmental process. Young Oksapmin children who know the standard set of numerals nonetheless compare body parts with respect to their physical similarity rather than with respect to their values as summations of correspondence relations. When comparing the values of symmetrical or asymmetrical body part pairs that were reached from a common point of origin (right thumb), the younger children incorrectly identified symmetrical body parts as equivalent in value, and when they correctly identified asymmetrical body parts as not equivalent, did not regularly identify which of the two represented the larger value. Older children produced accurate body part comparisons, regardless of the physical similarity of the body parts. A similar developmental process was manifested with the point of origin task. Younger children considered the same body part reached from two different points of origin (left and right thumbs) to represent the same value. In contrast, children in the older age group considered the value of a body part to be based upon its ordinal position in an enumeration. The findings from these tasks indicate that the Oksapmin children do not dissociate physical characteristics of their standard set of body parts from the use of these body parts as signifiers for correspondence relations. Over the course of development, children use the standard sequence of body parts to notate a progressive summation of correspondences.

The relations between children's acquisition of counting as a means to represent numerical information and their understanding of number conservation varied across the four studies. The relations reported in Studies 1 and 2 are consistent with findings from Western populations which indicate that children acquire the use of counting to compare and reproduce static sets prior to understanding number conservation. In contrast, there was no clear evidence that an understanding of counting preceeded the development of number conservation in Study 3 (although the trends were in this direction). It is plausible that the fact that concrete materials were used in the counting tasks in Studies 1 and 2 but not in Study 3 created figural supports which had a facilitative effect on children's use of counting to compare and reproduce number.

In summary, while the age norms for counting and conservation concepts are later in Oksapmin communities than those reported in the West, there are a number of similarities between Oksapmin and Western children's developing understanding of number. Oksapmin children, as do their Western counterparts, experience a pre-mediational phase in their acquisition of their cultural group's counting system. The generality of the pre-mediational phase suggests that it serves an important function in the early development of numerical symbolization. In particular, through incorporating a conventionally defined list of numerals into their own enumerative activities, children establish the pre-conditions for discovering the structure of their cultural group's system for numerical reference. The findings also indicated that, as in the West, Oksapmin children tend to transform their cultural group's standard set of numerals into their own notational system for number prior to achieving an understanding of number conservation.

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Table 1

Frequency Distributions of Children's Performance for Small
Set Size Reproductions and Comparison Over Age Group

Age Group	Task and Accuracy Level			
	Reproduction Task		Comparison Task	
	Inaccurate	Accurate	Inaccurate	Accurate
6-9 years	5	8	6	5
11-16 years	0	8	0	8

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Table 2

Frequency Distribution of Children's Counting Strategies for Intermediate
and Large Set Size Reproductions and Comparisons Over Age Group

Counting Task, Set Size, and Strategy Level

Age Group	Reproduction Task				Comparison Task			
	Set Size: Six		Set Size: Nine		Set Size: Six		Set Size: Nine	
	Pre-med-	Media-	Pre-Med-	Media-	Pre-Med-	Media-	Pre-Med-	Media-
	lational	tional	lational	tional	lational	tional	lational	tional
6-9 years	7	6	6	6	10	4	9	4
11-16 years	0	8	0	8	0	8	0	8

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Table 3

Frequency Distribution of Children's Conservation and Reproduction Counting
Strategy Levels for Intermediate and Large Set Sizes

Set Size and Conservation Level						
Counting Strategy	Set Size: Six			Set Size: Nine		
	No Conser- vation	Transi- tional	Conser- vation	No Conser- vation	Transi- tional	Conser- vation
Pre-						
Mediational						
Strategies	1	4	1	4	1	0
Mediational						
Strategies	1	5	12	3	6	8

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Table 4

Frequency Distribution of Children's Conservation and Comparison Counting
Strategy Levels for Intermediate and Large Set Sizes

Set Size and Conservation Level						
Counting Strategy	Set Size: Six			Set Size: Nine		
	No Conser- vation	Transi- tional	Conser- vation	No Conser- vation	Transi- tional	Conser- vation
Pre- Mediational Strategies	2	3	2	5	4	0
Mediational Strategies	0	5	11	2	4	8

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Table 5

Frequency Distribution of Children's Conservation and Counting Strategy Levels

Conservation Level	Counting Strategy			
	Comparison Task		Reproduction Task	
	Pre-Mediational	Mediational	Pre-Mediational	Mediational
No Conservation	2	15	3	14
Transitional	0	1	0	1
Conservation	0	8	0	8

Table 6

Frequency Distribution of Children's Performance on Asymmetrical and Symmetrical Body Part Comparison Tasks Over Age Groups

Task and Number Correct								
Age Group	Asymmetrical Body Part Comparisons				Symmetrical Body Part Comparisons			
	0	1	2	3	0	1	2	3
7-9 years	4	5	3	10	11	4	3	4
10-11 years	1	4	5	4	9	0	1	4
12-16 years	0	0	0	16	3	1	4	8

Table 7
Frequency Distribution of Children's Performance
on Point of Origin Task at Two Age Levels

Age Group	Number correct		
	0	1	2
7-9 years	11	5	4
12-16 years	1	6	8

Figure Caption

1. The conventional sequence of body parts used by the Oksapmin. In order of occurrence: (1) tipana, (2) tipnarip, (3) bumrip, (4) hAtdip, (5) hAtnAtta, (6) dopa, (7) besa, (8) kir, (9) towAt, (10) kata, (11) gwer, (12) nata, (13) kina, (14) aruma, (15) tan-kina, (16) tan-nata, (17) tan-gwer, (18) tan-kata, (19) tan-towAt, (20) tan-kir, (21) tan-besa, (22) tan-dopa, (23) tan-tipana, (24) tan-tipnarip, (25) tan-bumrip, (26) tan-hAtdip, (27) tan-hAtnAtta.

Culture, Counting, and Number Conservation

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The study reported here replicates, in three Papua New Guinea societies, research carried out with American children. In the U. S., the ability to use counting regularly precedes the ability to conserve number. In Papua New Guinea this relationship holds but the age of acquisition for both skills is delayed relative to Western norms. Variations in the nature of the indigenous counting system in the three societies has no apparent effect on the order of emergence of these skills.

The relationship between counting and number conservation has been studied by a number of investigators. A point of controversy has been whether children use counting as a means to mediate their quantitative activities before or after they develop an understanding of the conservation of number. Although Piaget (1) has argued that children's counting to mediate quantitative thought must necessarily develop after conservation, others have argued that there is an inherent relation between counting and conservation which leads to the earlier emergence of counting than an understanding of conservation (2, 3). Recent research has tended to support the latter position. For instance, Saxe (2) found that none of his 42 subjects (out of 66) who conserved number used a pre-quantitative counting strategy when they compared or reproduced sets of objects; however, among quantitative counters, roughly half failed to conserve number. In this American sample then, counting precedes number conservation but both developing processes appear almost simultaneously during a very narrow age range (4-6 years).

In Papua New Guinea, different cultural groups employ different types of counting systems to represent number, and within the country itself, people in different groups differ in the types of everyday quantitative activities in which they engage (4). Papua New Guinea therefore provided a particularly appropriate field site to test whether the sequence in the development of counting and conservation found in the West is due to a particular artifact of Western culture.

There are four different types of counting systems which people use in Papua New Guinea (5), of which we sampled cultural groups that used three of these types. Among the Oksapmin speaking people of the New Guinea Highlands (West Sepik) a body-part numeration system is used (6). Twenty-seven named parts

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of the upper body beginning with the right thumb are used in counting; the system has no base. Melpa-speakers of the Central New Guinea highlands use the fingers of both hands in a base-4/base-8 counting system (7). People on Ponam island in the Admiralty Archipelago do not use body-parts; the number names are unique and are organized into a base-10 system quite comparable to systems predominate in the West (5). Eighty-six children from these three groups were tested in their home villages. They ranged in age from 5 to 16 years (and had virtually no formal schooling). Children were given several trials each on conservation of number and reproduction and comparison counting tasks (8). Children from the Oksapmin communities used their indigenous system, although children from the Melpa and Ponam communities had sufficient contact with the West so that they employed the Western system in the tasks.

The results showed that in all communities children used counting, and that younger subjects tended to use "pre-quantitative" strategies (they counted, but did not use their counting to inform their numerical comparisons or reproductions) whereas older children used quantitative counting strategies (9). The major difference between the Papua New Guinea and American children tested previously (2) is that while 82 percent of the American 6-year-olds used quantitative strategies exclusively, this figure is not reached until age 8 in Papua New Guinea (10). Some children in all three societies conserved number, however, only one of the 33 subjects who succeeded in number conservation failed to use a quantitative counting strategy exclusively, and among quantitative counters roughly half (36/66) failed to conserve number. Although these results are perfectly consistent with the original American study (2), as Figure 1 shows, the period during which counting and number conservation

develops is considerably extended in Papua New Guinea. While the age effect is significant for counting (Chi square = 39.87, 4 df, $p < .001$) for conservation it is not (Chi square = 2.49, df = 2, n.s.). The trends in Figure 1 also suggest that while children begin to use quantitative counting strategies before they understand number conservation in the Papua New Guinea communities, the margin between the age norms for the acquisition of these skills is considerably wider in these communities than in the United States. It is also of some interest to note that in another Papua New Guinea study in which conservation of length was the dependent variable and a full age range was tested (6 years to adult) the ability to conserve never exceeds 50% in a population of individuals who have had no formal schooling. Thus, while the ability to use a conventional system to represent systems for number representation may be an early emerging universal feature of cognitive development, conservation at least as it is assessed in Piaget's classic assessment procedure, is probably not (12).

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10. It is important to note that among the Oksapmin sample ages were approximated by comparing Oksapmin children of known age with children of unknown age. Children's ages in the other two samples were determined by missionary birth records.
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12. Approximately half of the Oksapmin-speaking subjects were administered Piaget's tests for conservation of mass (clay) and seriation; all of the Ponam speakers and half the Melpa speakers were given the seriation task. Success on these concrete operational tasks never exceeded 5% in any sample.

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13. This research was supported by a grant from the Papua New Guinea Department of Education and a grant from the national Institute of Education.

Geoffrey B. Saxe

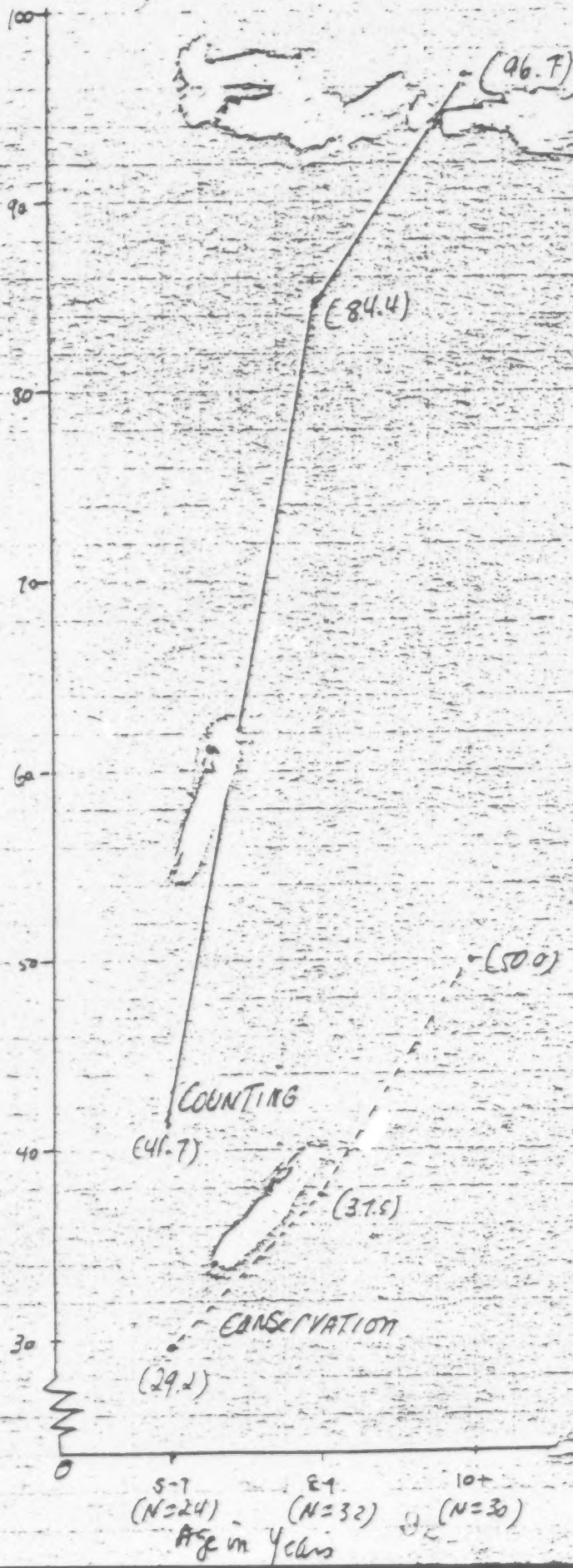
C.U.N.Y. Graduate Center
33 West 42nd Street
New York, NY 10036

David F. Lancy

Department of Psychology
University of California
Los Angeles, CA 90024

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Piagetian Theory and the Atypical Case:
An Analysis of the Developmental Gerstmann Syndrome

Geoffrey B. Saxe

The Graduate School & University Center
of the City University of New York

Sandra Shaheen

The Children's Hospital Medical Center, Boston

Abstract

This paper presents an analysis of a particular form of atypical cognitive development from the point of view of Piaget's normative developmental theory. Two 9-year-old boys are described who performed in the normal range on measures of I.Q., language ability and reading. The boys were unable, however, to acquire elementary numerical skills and also manifested other specific cognitive deficits associated with the Gerstmann syndrome. The results of additional testing on a battery of Piagetian tasks indicated that neither child had advanced to the Piagetian stage of concrete operations. These findings have implications both for Piaget's theory and for understanding the nature of the atypical development. With respect to atypical development, the findings indicate that children's acalculia is not an isolated deficit, but rather that the general delay in the stage transition inhibited the children's development of numerical skills. With respect to Piagetian theory, the finding that the boys had not achieved numerical skills, but had acquired reading skills, suggests that whereas the acquisition of the former are closely linked to Piaget's stages of operational development, the acquisition of the latter are not directly dependent upon stages of operational development.

Piagetian Theory and the Atypical Case:
An Analysis of the Developmental Gerstmann Syndrome

Methods for studying human development are divisible into two types. One focuses on the individual case and seeks to describe unique patterns of development (idiographic), and the other focuses on the general case and seeks to discover universal principles of development (nomothetic). For the most part, idiographic methods have been applied to instances of pathological development in order to explore particular forms of pathological deviation, whereas nomothetic methods have been applied normal populations in order to advance our understanding of invariant characteristics of developmental change. Allport (1937) has perhaps most forcefully argued that psychology must rely on an orchestration of both idiographic and nomothetic approaches to reach an understanding of human behavior. That is, our understanding of the particular case must be grounded in a general theory of development, and, theoretical formulations must be able to incorporate instances of atypicality and individuality.

This paper presents an attempt to coordinate an idiographic analysis of cases of atypical cognitive development (number disorders in the developmental Gerstmann syndrome) with a normative formulation about general principles of development (Piaget's analysis of cognitive development during middle childhood) such that both the atypical case and the normative formulation receive some clarification. To this end, the present paper is organized into three sections. First, some background about the development of atypical numerical concepts and the relevance of

Piaget's normative formulation are presented. Second, an idiographic description is presented of two children who manifest an atypical development of numerical concepts, as well as other symptoms which are associated with the developmental Gerstmann syndrome. Third, the cases are discussed with respect to their implications for a modification of the normative formulation, as well as with respect to the way in which the normative formulation clarifies aspects of the etiology of the syndrome itself.

NUMBER DISORDERS AND A NORMATIVE APPROACH

In one of the better known reports of atypical numerical cognition, Gerstmann (1930) claimed that a syndrome of four cognitive deficits, including acalculia (an inability to understand arithmetical relations) was associated with a lesion of the parietal cortex of the dominant hemisphere. Besides acalculia, the other deficits of the syndrome include agraphia (the inability to write), finger agnosia (the inability to recognize one's own fingers), and left/right disorientation. Although there is considerable controversy as to whether Gerstmann's tetrad of deficits actually can be considered a "syndrome" (cf. Benton, 1977; Benton, 1961; Kinsbourne and Warrington, 1962; Heimbürger, Demyer, and Reitan, 1964; Strub and Geschwind, 1974), the neurological literature is replete with descriptions of patients with isolated mathematical disorders. Moreover, similar mathematical deficits have been reported in children who otherwise show normal levels of intellectual functioning (e.g., Benton and Geschwind, 1970; Cohn, 1968; Kinsbourne and Warrington, 1963; Kosc, 1974; Pirozzolo, and Rayner, 1978).

Piaget's theoretical and empirical work on cognitive development provides a possible means of exploring the underlying cognitive basis of Gerstmann children's acalculia. Piaget (1970) demonstrated that between

4 and 7 years of age children's ability to solve simple logical problems not only improves, but the nature of children's thinking about the problems actually undergoes a qualitative change (Piaget, 1970). Piaget found that one of the salient characteristics of the 4-year-old's thinking is that it is irreversible. The child focuses on static states of reality and cannot understand relations between states. For example, the child believes that the quantity of a set of beads changes if the beads are pushed together or spread apart. In contrast, by 7 years of age, the child begins to reason in terms of concrete logical or reversible operations, and it is this developmental change that provides the cognitive basis for the child's acquisition of fundamental arithmetic concepts.

On the basis of Piaget's normative work,

the presence of atypical mathematical behavior among Gerstmann children who are 7 or 8 years of age and above and are of normal intelligence is a puzzle. It may be that acalculia is truly an isolated deficit and that its occurrence is unrelated to any general cognitive-developmental disorder. On the other hand, it may be that acalculic children experience a developmental delay in the stage transition. If such a delay in the stage transition occurs, it would place enormous constraints on the development of logico-physical concepts, such as arithmetic, but may leave the development of other cognitive skills which are not directly dependent on the stage transition relatively unimpaired (e.g., certain aspects of reading). Thus, such a population of children may achieve approximately normal scores (yet unusual test profiles) on standard measure of I.Q., but at the same time these children would demonstrate severe deficits in their understanding of arithmetic.

In the section to follow, two 9-year-old boys are presented who both show the tetrad of deficits associated with the developmental Gerstmann syndrome. On the battery of neuropsychological tests, both children demonstrated I.Q.'s in the normal range, no serious language problems, and no serious visual-perceptual impairments. Moreover, one of the children demonstrated reading skills well above age level. Our concern was first to document the children's atypical mathematical behavior and the other deficits associated with the Gerstmann syndrome, and then to determine the children's progress in the stage transition from preoperational to concrete operational reasoning. In this way we would assess whether the atypical arithmetic behavior was an isolated impairment, unrelated to the normative stage transition in logical reasoning, or whether these children's atypical mathematical behavior could be attributed to a developmental delay in the stage transition.

THE CASE HISTORIES

Case 1

P. was an 8 11/12 old caucasian boy who was referred for neuropsychological testing by his teachers to obtain new educational insights. At the time of testing, P. was in his third year in a ungraded classroom in a college-sponsored learning center. He was doing well in all subjects except mathematics where his skills were at a pre-arithmetic level according to formal and informal screening. In contrast, P.'s reading and comprehension skills were at or above grade level and P. was a self-taught reader by 4 years of age.

History

P. was the youngest of three children, delivered two weeks early by Caesarian section and weighing 8 lbs. 6 ozs. P.'s father was a consulting engineer and his mother was a nurse. No learning or neurological difficulties were reported in the parents' or siblings' histories, and all were right-handed. P.'s motor and speech milestones were delayed: he was not walking until 22 months and he did not begin to use language until 36 months. At time of testing, P. demonstrated inconsistent hand preference in the testing situation although Dynamometer trials indicated stronger left hand grasp. Past History was negative for significant head injuries, major surgery, or other neurological symptomology. An audiological examination at age 8 years indicated normal peripheral hearing and no specific deficit in central auditory processing. He was described by his parents and his teacher as "immature." That is, he was highly distractable, had temper tantrums, and his peer experiences were primarily limited to parallel rather than interactive play.

Neuropsychological Testing

The aim of neuropsychological testing was to determine whether P.'s learning difficulties were attributable to a specific system dysfunction (e.g., language impairment). To this end, we administered language, perceptual, motor, and praxis tests. These tests included in the WISC-R, an expressive-receptive language battery, a gross and fine motor inventory, and an academic achievement test.

P. scored in the normal range on the WISC-R, despite a depressed verbal subscore due to low performance on Arithmetic and Digit Span (reversed condition) subtests (Full scale IQ: 100, Verbal IQ: 108, Performance

IQ: 91). Poor performance on constructional tasks (particularly Block Design) and the number symbol association task (Coding) accounted for a depressed performance score. Many subtests involving language, perceptual skills, and verbal abstract reasoning were performed in the average or superior range.

In order to further evaluate P.'s language abilities we administered additional test inventories covering the following areas: verbal analogy, complex syntax, reading, and auditory memory. P. performed at age level across these measures, although, some articulation and sequencing difficulties were noted. For instance, although auditory memory for logical passages, sentences, and forward digits were good for P.'s age level, P. was unable to reverse digit order. P. demonstrated age appropriate levels on reading and comprehension measures. In sum, P.'s, receptive language skills exceeded his expressive abilities, and his sequencing and articulation difficulties were his most notable expressive language problems.

P.'s motor milestones were severely delayed according to Lincoln-Ozeretsky and Denver Developmental norms (below a five year level). His pencil grasp and fine motor dexterity were exceptionally poor. In contrast, abilities in the visual-perceptual modality were strong. The Ayres Visual Figure-Ground Discrimination test and a visual analysis task from the WISC-R were used as measures of visual-perceptual skill (no motor component was involved in these tasks).

CASE 2

B. was a 9 1/2-year-old caucasian boy who, had made few academic gains. after three years in an ungraded classroom. At the time of testing, teachers considered his greatest weaknesses to be short attention span,

poor arithmetic, and poor visual motor skills. Behaviorally he was described as an active, cooperative boy who did not present significant behavioral management problems.

History

B. was the youngest of three children in an intact family, delivered after a full-term uncomplicated pregnancy. Developmental milestones were delayed: he did not walk until 23 months and did not begin to use words until 25 months. Motor milestone since that time have been delayed and at the date of testing he was participating in a physical education program and receiving fine motor therapy. There was no family history of learning or neurological difficulties. B. displayed mixed hand and foot dominance, and left eye dominance in a family of right handers. Past history was negative for significant postnatal head injury, major illnesses or surgery. At age 22 months, neurologists diagnosed a hypotonic syndrome probably dysgenetic in origin. An EEG and Audiological exam were performed at age 8 years and were normal.

Neuropsychological Testing

As in the case of P., the aim of neuropsychological testing was to determine whether B.'s cognitive difficulties were attributable to a specific system dysfunction (e.g., language impairment.) The tests included the WISC-R, an expressive-receptive language battery, a gross and fine motor inventory, and an academic achievement test.

B.'s performance on 6 of 10 WISC-R subtests was within the average range, and subtests requiring verbal abstract reasoning, visual perceptual abilities, and language skills were performed at age level (full scale I.Q.: 81, verbal I.Q.: 82, performance I.Q. 81). B.'s spontaneous speech was fluent, appropriate for age in vocabulary, structure, and tone. Testing indicated that B. had average abilities

in receptive language skills, general information, comprehension, and in recalling logical passages in sequence and preserving the meaning.

B.'s motor milestones on the Lincoln Ozeretsky and Denver Developmental were significantly delayed (between the 4-5 year level). His pencil control and fine motor skills were markedly poor. On a test of visual Figure-Ground discrimination without a motor component, P. performed at age level. Despite visual analytic competency and a history of recognition reading before entering school (of street signs, simple information signs in public places), his reading abilities were only at the middle of the first grade level.

TESTING PROCEDURES

In order to assess the Gerstmann syndrome and the children's progress in the stage transition from pre-operational to concrete operational thought, we administered two sets of measures to each child: (1) tasks designed to assess the tetrad of Gerstmann symptoms including left/right orientation, finger gnosis, graphic skills, and mathematical behavior; and (2) tasks designed to assess children's progress in the stage transition from pre-operational to concrete operational thought including four Piagetian tasks: number conservation, class inclusion, continuous quantity conservation, and seriation.

Gerstmann Syndrome Tasks

Left/right orientation. The interviewer asked the child to indicate his left and right hands, and then to carry out instructions involving left/right orientation in crossed and uncrossed conditions.

Finger gnosis. A series of finger localization tasks adapted for children were used to assess finger gnosis (Kinsborne and Warrington, 1967). In the first task, the interviewer blindfolded the child and asked him to point to the fingers that the interviewer touched. In the second task, the interviewer touched two points on the child's fingers, either on the same or adjacent fingers, and the child was required to say whether one or two fingers were being stimulated. In the third task, the interviewer presented the child with four wooden blocks (as illustrated in Figure 1). The interviewer asked the child to close their eyes and then the children's fingers were molded around one of four blocks. The child was asked to open their eyes and without looking at the block in his hand, to pick out a matching block on the table.

Graphic Skills. The interviewer asked the child to write his name with preferred and non-preferred hand. In addition, each child was administered the Beery Visual Motor Integration Test (VMI) which is an age-normed test of ability to copy simple and integrated geometric forms. Writing and spelling difficulties were evaluated according to norms of the Wide Range Achievement Test. Finally, each child was asked to make a representational drawing of a person, ordinarily accomplished by 3 or 4-year-olds.

Mathematical behavior. Each child was administered two sets of numerical tasks.

(a) Rote knowledge of the number system. The following tasks were designed to assess the child's rote knowledge of the number system.

Counting aloud. The interviewer asked each child to count aloud as far as he could. Children were stopped once they reached 21. Identifying a numeral between two others. The interviewer asked the child, "What's a number in between 11 and 14?, 28 and 31?, 13 and 17?" Writing numerals. The interviewer read the following list of fourteen numbers. Following each presentation, the child was asked to write the numeral on a piece of paper: 63, 9, 36, 15, 37, 3, 13, 93, 14, 16, 34, 7, 35, 96. Reading numerals. The interviewer asked each child to read a list of fourteen numbers typed on a large card. The same numbers were used on this task as in "Writing numerals."

(b) Applying the Rote System. The following tasks were designed to assess children's ability to apply the rote number system to solve elementary number problems: Counting accuracy. The interviewer presented the child with a set of 5" x 9" cards containing arrays of dots. Upon the presentation of each card, the interviewer asked the children, "How many dots are there?" The cards contained 4, 9, and 11, one-fourth inch diameter dots, in both linear and random arrays. On each card the dots were easily visible and distinguishable from one another. Counting to compare arrays. The interviewer presented the child with a 5" x 9" card containing a linear array of ten 1/4" red dots that extended 2 inches more than a parallel array of eleven blue dots. The interviewer asked, "Are there more red or blue dots?" "How do you know?"

Counting to reproduce arrays: Drawing Task.

The interviewer presented a linear array of ten one-inch diameter circles

pasted to an 8" x 11" piece of paper, and asked the child to draw the same number on a 3" x 5" rectangular card. Counting to reproduce arrays:

Remote placement task. The interviewer presented nine 1/4" diameter beads underneath the child's table, and asked the child to put out just the same number on top of the table from an available set. The placement of the model collection was such that the child could not see the model while constructing the copy. Numerical calculations. The interviewer presented each child with the following set of addition and subtraction problems: $1 + 3 =$, $5 + 4 =$, $8 - 2 =$, and $5 - 3 =$.

Concrete Operational Tasks

We used the following set of tasks to assess children's progress in the transition from pre-operational to concrete operational thought. As noted, typically children achieve an understanding of these tasks by 7 or 8 years of age.

Number conservation task. The experimenter presented two rows of ten beads aligned in one-to-one correspondence. The child was asked if the two rows contained the same number. One of the rows was then spread apart and the child was asked "Do the rows have the same number, or does one have more?" "How do you know?"

Class inclusion task. The experimenter presented the child with a set of ten wooden beads eight of which were blue and two of which were

red. The experimenter asked "What are these beads made of?" If the child did not respond "wood" the child was told that the beads were each made of wood. The experimenter then asked, "Are there more wood beads or more blue beads?" "How do you know?" If the child did not respond with the correct answer and an appropriate explanation, the child was presented with an additional set of questions to check the child's comprehension of the task: "Are there more blue beads or red beads?" "If I gave you the red beads, what would be left in my pile?" "If I gave you the blue beads, what would be left in my pile?" "If I gave you the wooden beads, what would be left in my pile?" "Are there more wood beads or more blue beads?"

Seriation task. The experimenter presented the child with ten sticks, ranging in length between 4 and 7 inches. The experimenter asked the child to "put the sticks in order from the very shortest to the very longest." The child's procedure for constructing the series as well as the final constructions were recorded. If the child succeeded in constructing a series, he was given four additional sticks, one by one, to insert in the appropriate location within the series.

Liquid quantity conservation task. The interviewer presented the child with two beakers (A and B) of the same dimensions, filled to the same level, and asked, "Do these have just the same amount?" After the child agreed, the water in one of the beakers was poured into another beaker with a smaller cross (A'), and the interviewer asked, "Do both of these glasses (A and A') have just the same amount, or does one have more water?" "How do you know?"

RESULTS

Table 1 contains a summary description of children's performance on the tasks designed to assess the Gerstmann syndrome, numerical abilities, and the Piagetian concrete operations.

Insert Table 1 about here

Gerstmann Tasks

Both children present the classical symptomatology of the developmental Gerstmann Syndrome. Neither child passed the subtests on left/right orientation, and both children had difficulty on the finger gnosis tasks. Both boys also performed poorly on all measures of graphic skills. Results on the Ares Figure-Ground Discrimination test indicate that the children's dysgraphia cannot be attributable to poor visual perception; for each boy, visual perceptual ability was nearly one standard deviation above the mean for their respective ages.

Number Ability Tasks

Both children's performance on the number abilities tasks were very poor.

Knowledge of the rote system. Although both children could count by rote, both had difficulty naming a number that occupies an interval between two others; moreover, both children had difficulty reading and writing numerals. For instance, B. wrote three out of fourteen numerals correctly to dictation, and each of these were single digit numerals. His

mistakes were of the following order. He wrote "63" as "32", as he articulated "three, six". He wrote "15" as "51", "35" as "51", and "14" as "41". Reading numbers was also extremely difficulty for B. He responded correctly on only six out of fourteen trials. His mistakes were of the following order: He read "63" as "three and nine", "37" as "seventeen", and "35" as "thirteen and fifteen".

P. had difficulty writing, but not reading numbers. Although P. wrote ten out of fourteen numbers correctly, five of his successes were a product of trial and error. For instance, when asked to write "37" he conducted the following labored procedure. He wrote a "6" that he crossed out but then wrote "6" again. He then crossed out the second "6" and wrote "5". He then crossed out the "5" and correctly wrote "37". Similarly, when asked to write "34" he first wrote "4", then "46", then crossed out the "46" and wrote "6". He then crossed out the "6", and wrote the stimulus as a reversal, "43". P. also tended to reverse digits and write numbers in mirror-image forms, although he frequently monitored these errors, and in the end, achieved a correct result. For instance, he began to write thirteen as "E" then corrected it to "13"; he did the same with ninety-three. Similarly he wrote "16" and "19" in reverse orders and corrected both of them.

Applying the rote system. Both children also demonstrated serious difficulties when they used the number system to solve elementary number problems. For instance, although both children counted collections of four elements accurately, both were consistently inaccurate when they counted larger collections of 9 and 11 objects. Their inability to use counting as a "symbolic tool" to manipulate number was apparent on the tasks that assessed how children used counting in numerical comparisons and reproductions

(See Saxe, 1977, for an analysis of normal children's performance on these tasks). Not only were they inaccurate in their counting, but both children could not use their counting to produce numerical comparisons and reproductions. For instance, although both children counted in the numerical comparison task, B. counted both rows continuously as if they were one. B. counted both rows separately, but then based his numerical evaluation on the spatial extent of the arrays, not on his counting. Although the children showed some familiarity with the arithmetic symbols, neither child could solve the single digit addition and subtraction problems.

Concrete Operational Tasks

Children's performances on the concrete operational tasks indicate that neither had progressed to the stage of concrete operational thought. Both children believed that changing the spatial configuration of either a discontinuous quantity (as in the number conservation task) or a continuous quantity (as in the liquid conservation task) actually changed its amount. Moreover, neither child could order the series of sticks from the shortest to longest (instead both tended to produce a collection of seriated couplets or triplets), and, on the class inclusion task, both children failed to understand that there were more wooden beads in the pile than red beads.

Discussion

Previous discussions of the developmental Gerstmann syndrome have characterized children's acalculia (as well as their left/right disorientation, finger agnosia, and agraphia) as an isolated cognitive deficit that co-occurs with I.Q. scores in the normal range. Both children presented in this report had test profiles consistent with this description. An analysis of these children from the point of view of Piagetian theory, however, not only

provides some new insights about the specificity of these children's cognitive deficits, but, reciprocally the nature of the children's atypicality serves to inform some aspects of Piaget's normative developmental theory.

With respect to the implications of the theory for the children's atypicality, additional testing on the Piagetian tasks showed that the children's acalculia was not an isolated deficit at all. On the contrary, by 9 years of age, neither child had begun the stage transition from pre-operational to concrete operational reasoning. This finding implies that the children's acalculia is only a single manifestation of a quite general cognitive-developmental delay.

With respect to the implications of the atypical cases for developmental theory, the present findings indicate that although children were unable to acquire elementary numerical skills, they had achieved other intellectual skills such as some facility with reading. The finding that the acquisition of such skills as arithmetic and reading were dissociated in the present cases suggests that, whereas the former are dependent on Piaget's stages of operational development, the child's acquisition of the latter are not directly dependent upon the Piagetian stages of operational development.

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Table 1

Summary of Children's Performances Across the
Gerstmann, Number, and Concrete Operational Tasks

Gerstmann Tasks

	<u>P.</u>		<u>B.</u>	
L-R Orientation	failed		failed	
	failed		failed	
Finger gnosis	left	right	left	right
Task 1	1/20	10/20	2/20	4/20
Task 2	<u># correct</u>		<u># correct</u>	
	<u># trials</u>		<u># trials</u>	
Task 2	0/10	1/10	0/10	0/20
Task 3	failed		failed	
Wood Block	failed		failed	
Graphic Skills				
Visual Motor Integration	5 yrs., 3 mo.		4 yr., 6 mo.	
Spelling (WRAT)	1.0 grade level		1.0 grade level	
Name Printing	dominant hand <u>85 sec</u>		<u>65 sec</u>	
	non-dominant hand <u>90 sec</u>		<u>75 sec</u>	
Ayres Fig. Gr. Vis Discrim.	+0.9 SD.		+1.0 SD.	
Acalculia				
WISC-R Arith.	SS = 4		SS = 2	

Table 1 (Continued)

Number Tasks	Rote Numeration Abilities		P.	B.
Applying the Rote System	Counting aloud to 21		+	+
	Identifying a numeral between two others		3/4	0/3
	Reading numbers		13/14	6/14
	Writing numbers to oral dictation		10/14	3/14
	Counting accuracy 4 elements		2/2	2/2
	Counting accuracy 9, 11 elements		0/4	1/4
	Counting to compare and reproduce arrays		Fluctuates between pre-quantitative and trial and error strategies	Fluctuates between pre-quantitative and trial and error strategies
	Arithmetic calculations		1/4	0/4
Piagetian Concrete Operational Tasks	Number conservation		-	-
	Class Inclusion		-	-
	Liquid Conservation		-	-
	Seriation		-	-

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Section III.

Children's Understanding of Conventional and Logical Properties
of Counting

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Correspondences and Conventions: A Developmental Analysis of

Formal Conceptions of Counting

Geoffrey B. Saxe, Stephen Sicilian, and Irvin Schonfeld

The Graduate Center of the City University of New York

Running Head: Correspondences and Conventions

Correspondences and Conventions

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Abstract

The three studies of this report concern developmental changes in children's formal understanding of counting and the relation of these changes to children's understanding of number conservation. In study 1, 50 4-, 8-, and 11-year-olds judged the relative adequacy of the counting of 2 finger puppets, one who violated logical features of counting (established many-to-one instead of one-to-one correspondences between numerals and objects) and the other conventional features of counting (used letters as numerals). With increasing age, children correctly identified the unconventional puppet as the "good" counter. In Study 2, it was found that 6-year-olds who had a prior history of experience with multiple systems of signification (bilinguals) tended to develop a formal understanding of counting prior to their monolingual peers. Moreover, whereas bilinguals tended to develop a formal understanding of counting prior to number conservation, monolinguals tended to develop an understanding of number conservation prior to counting. In Study 3, it was found that when the unconventional numeral list was changed from the alphabet to a set that was not embedded in other functional contexts for monolingual children (Chinese numerals), monolingual's performance significantly improved.

Correspondences and Conventions: A Developmental Analysis of
Formal Conceptions of Counting

Most of the recent research on the development of counting focuses on children's use of counting to represent number (e.g., Gelman & Gallistel, 1978; Klahr & Wallace, 1976; Saxe, 1977, 1979, Schaeffer, Eggleston, & Scott, 1975). No research to date concerns the child's understanding of the organization of counting as a conventional symbol system for numerical representation. The purpose of this paper is to present an analysis of developmental changes in children's understanding of the formal organization of counting systems and the developmental relation between a formal understanding of counting and number conservation.

SOME FORMAL ASPECTS OF COUNTING SYSTEMS

The organization of counting systems has features that are both conventional and logical. The particular set of numerals of a counting system is a conventional aspect of counting. These elements are arbitrary and vary from one culture to another. In contrast, the one-to-one correspondences that a subject produces between numerals and objects is a logical feature of counting systems. This feature of counting systems is universal, since, for any numeral to have a cardinal significance, it must refer to one-to-one correspondence relations with objects. A formal conception of counting would entail knowing that any set of ordered elements, in principle, can be used as numerals in counting, if logical correspondences are not violated between numerals and objects. It is this definition of a formal conception of counting that will guide the analysis to follow.

THE DEVELOPMENT OF A FORMAL UNDERSTANDING OF COUNTING
AND ITS RELATION TO NUMBER CONSERVATION

Piaget's analysis of the development of number conservation indicates that children's understanding of one-to-one correspondence relations undergo qualitative changes over age. Since counting has its cognitive basis in one-to-one correspondence relations, children's understanding of the formal organization of counting systems should undergo qualitative changes as well. An examination of Piaget's 3 stage description of children's developing knowledge of number conservation leads to some specific hypotheses about developmental relations between these two kinds of conceptions.

In the classical assessment procedure for number conservation, the child is presented with a linear array of objects and asked to put out the same number from an available set. At the first stage, children do not produce one-to-one correspondence relations between the model and copy; instead, children tend to match the endpoints of their copy with the model. At the second stage, children do produce one-to-one correspondence relations. However, once one of the arrays is displaced in the conservation task, children believe that the number changes. During the third stage, children understand that the one-to-one correspondence relations are conserved over the spatial transformation.

It will be argued on the present formulation that qualitative changes in children's formal understanding of counting and number conservation should occur partially independently of one another. That is, some children

will achieve a formal understanding of counting prior to number conservation whereas others will achieve an understanding of number conservation prior to counting. The rationale for this hypothesis is that although both counting and number conservation each necessarily require an understanding of the numerical significance of static one-to-one correspondence relations, Piaget's second stage of his three stage sequence, each of these concepts also entail additional and partially independent conceptual abilities. Number conservation, unlike counting, requires the child to understand the invariance of one-to-one correspondence relations over spatial transformations. A formal understanding of counting, unlike number conservation, requires an appreciation of the arbitrary or conventional nature of numerical symbols. Since these sets of differences should raise independent kinds of conceptual difficulties for children, there should be no necessary sequence in the emergence of these concepts in development. Once children achieve an understanding of the numerical significance of static one-to-one correspondence relations (required for either conception) children may achieve the understanding of one concept independent of the other.

In order to trace developmental changes in children's understanding that counting consists of conventional and logical features, and the relation of this understanding to children's conception of number conservation, a series of studies were conducted. In Study 1, a task procedure was devised to analyze developmental changes in children's conceptions of the formal organization of counting systems. —————→

In the task procedure, children were required to evaluate the relative adequacy of the counting of two finger puppets, one who violated conventional features of our counting system (used a different standard set of numerals) and the other who violated logical features of the counting process (established many-to-one instead of one-to-one correspondences between numerals and objects).. In Studies 2 and 3 the same task format was used to investigate factors hypothesized to influence children's understanding of counting as well as the developmental relation between children's understanding of counting and number conservation.

STUDY 1

Method

Subjects

A total of 50 children were interviewed. These included 18 4-year-olds, 16 8-year-olds, and 16 11-year-olds. The children were from middle class schools.

Materials

Bert and Ernie finger puppets, manufactured by Child Guidance Toys, were slightly altered for the purpose of the study. A capital "A" was drawn on the back of the head of the Bert puppet and a large number "1" was drawn on the back of the head of the Ernie puppet. Six white cardboard rectangles, 13 cm x 30 cm were also used. Either three or six of a homogeneous set of fruit decals (apples, oranges, or pears)

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were pasted on each of the cardboard rectangles in a straight line. The decals were each about 3 cm wide and were placed 6 mm apart from each other.

Procedure

Preliminary screening. Every child was asked to count (recite the numerals) to ten and recite the alphabet. We included only those children who could recite numerals to ten and who could recite the alphabet up to the letter f. All of the children met the criteria for inclusion.

Introduction to counting conditions. The child was presented with the Bert and Ernie finger puppets and told "This is really Ernie-1 and this is really Bert-A (the interviewer turned the puppets around to show the child their labels, and then continued the story). Ernie-1 comes from down the street where they use numbers to count, but Bert-A comes from another country far away where they use letters to count." The interviewer questioned the child in order to determine if the child understood the introductory story. If the child did not understand the introduction, it was repeated and the interviewer checked the child's comprehension again.

Counting conditions: set size three. (a) No violation condition. The interviewer told the child the following: "One day Ernie-1 and Bert-A were walking along. They saw a store and went in." The interviewer presented the picture of three apples, and asked the child to identify them. (Ernie-1 then says:) I WONDER

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HOW MANY THERE ARE. I'LL COUNT. THERE ARE 1, 2, 3 (counts with his nose). THERE ARE 3. (Bert-A says:) NO, THAT'S NOT RIGHT, THAT'S NOT RIGHT AT ALL. THERE ARE A, B, C (counts with his nose). THERE ARE C. The interviewer then asked the child, "What do you think? Did both Bert-A and Ernie-1 count right, or did one count right and the other count wrong?" After the child responded, the interviewer asked, "How do you know?" and additional probe questions.

(b) Numeral repetition condition. The interviewer told the child, "Ernie-1 and Bert-A walked further on in the store." The interviewer presented the picture of 3 oranges, and asked the child to identify them. (Ernie-1 then says:) I WONDER HOW MANY THERE ARE? I'M GOING TO COUNT THEM. THERE ARE 1, 1, 2. THERE ARE TWO. (Bert-A says:) NO, THAT'S NOT RIGHT, THAT'S NOT RIGHT AT ALL. THERE ARE A, B, C. THERE ARE C. (The interviewer then asked:) "What do you think? Did both Bert-A and Ernie-1 count right, or did one count right and the other count wrong?" After the child responded, the interviewer asked, "How do you know?" and additional probe questions.

(c) Numeral omission condition. The interviewer told the child "Ernie-1 and Bert-A walked further on in the store. (The interviewer presented the picture of the 3 pears and asked the child to identify them. Ernie-1 then says: I WONDER HOW MANY THERE ARE? I'M GOING TO COUNT THEM. THERE ARE 1, 2. Bert-A says: NO, THAT'S NOT RIGHT AT ALL. THERE ARE A, B, C. THERE ARE C. (The interviewer then asked,)

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"What do you think? Did both Bert-A and Ernie-1 count right, or did one count right and the other count wrong?" After the child responded the interviewer asked, "How do you know?" and additional probe questions.

Counting conditions: set size six. The three conditions administered with set size six were identical to the conditions administered with set size three. In the numeral omission condition, the Ernie-1 puppet miscounted by counting only four of the six pears, skipping the second and fifth pear. In the numeral repetition condition, Ernie-1 repeated the numerals "2" and "3" twice each.

Administration Procedures. Every child was assessed on the set size three and set size six test batteries. Set size was counterbalanced and within each set size, numeral repetition and numeral omission conditions were counterbalanced as well. Each no violation condition, however, was always administered prior to the violation conditions. The 8- and 11-year-olds were administered both batteries in the same session. Because they were more likely to fatigue as a result of the testing, the 4-year-olds were administered each of the test batteries on different days. However, no two testing sessions were more than one week apart, and any child who was tested on different days was introduced to the puppets before each of the testing sessions.

Due to a problem in sustaining some of the young children's attention, a few of the children's protocols were not complete. Two 4-year-olds did not complete the numeral omission - six condition, and one did not complete the no violation - six condition.

Scoring procedures. On the basis of children's responses to the counting conditions, we found that two scoring systems were necessary, one for the no violation conditions, and another for the violation conditions (numeral repetition and numeral omission).

The scoring system for the no violation conditions consisted of two levels. Level I was defined as the judgment that Ernie-1 counted correctly and that Bert-A counted incorrectly. Level II was defined as the judgment that both Ernie-1 and Bert-A counted correctly.

The scoring system for the violation conditions (numeral repetition and numeral omission) consisted of two levels which were parallel to the no violation scoring system, but also included a transitional level. For the violation conditions, Level I was defined as the judgment that Bert-A counted incorrectly and that Ernie-1 counted correctly. The transitional level was defined as the judgment that both puppets counted incorrectly. Level II was defined as the judgment that Bert-A counted correctly and that Ernie-1 counted incorrectly.

Results

Table 1 contains the distribution of children's levels of responses over age on the no violation conditions for set sizes 3 and 6. With increasing age, children's responses shifted from Level 1 to Level II for both set sizes (set size 3, $\chi^2_{(2)} = 17.95$, $p < .001$, set size 6, $\chi^2_{(2)} = 11.62$, $p < .01$). Table 2 contains the distribution of children's levels of responses on the violation conditions (numeral repetition and numeral omission). As in the case of the no violation condition, with increasing age, children's responses shifted from Level 1 or transitional responses to Level 2 responses on both the numeral repetition conditions (pooling Level 1 and transitional responses, set size 3, $\chi^2_{(2)} = 11.99$, $p < .01$, set size 6, $\chi^2_{(2)} = 8.79$, $p < .02$) and the numeral omission conditions (set size 3, $\chi^2_{(2)} = 11.99$, $p < .01$, set size 6, $\chi^2_{(2)} = 11.76$, $p < .01$). Sign tests revealed no differences in performance due to set size for any of the counting conditions at any age level. Sign tests indicated that at 8 years of age, more children achieved Level 2 on the numeral repetition and numeral omission conditions than on the no violation conditions for set

size 3 (numeral repetition/no violation, $N=5$, $x=0$, $p < .04$, numeral omission/no violation, $N=5$, $x=0$, $p < .04$). Since the no violation conditions were not counterbalanced with the violation conditions, these effects may have been due to the order of administration rather than the nature of the task conditions.

Insert Tables 1 and 2 about here

Discussion

The results of this study indicate, as predicted, that children's understanding of the formal organization of counting systems undergoes a qualitative change over development. Young children believe that a counting system must contain our list of numerals and no others, and therefore judge that Bert-A counted incorrectly. With increasing age, children understand that counting is defined by logical operations of one-to-one correspondence rather than the particular numerals that are used to notate correspondence relations, and therefore judge that Bert-A's counting was adequate and, when appropriate, that Ernie-1's counting was inadequate. We found that this age trend occurred across all six conditions that were administered, and therefore, it seems to be a general developmental process in the child's acquisition of counting.

Several aspects of children's performances are suggestive about basic developmental processes involved in the formal understanding of counting. Ostensibly the findings that there was no difference across conditions which was attributable to set size is not consistent with the previous studies that do report such differences in children's numerical abilities (e.g., Gelman, 1972;

Gelman & Tucker, 1975; Gelman & Gallistel, 1978; Winer, 1974). It is important to note, however, that the previous studies focused on children's ability to encode or extract numerical information and were not concerned with analyzing the child's understanding of the structure of the counting system. This is an important difference which we believe explains the child's consistency of performance across set sizes in our study. That is, the configuration of a collection may serve as an aid or perceptual support to codify number for small, but not large, arrays of objects. Since the tasks in the present study did not require the child to codify number through counting, but rather to demonstrate an understanding of the counting system, we observed no differences across set sizes in our tasks.

Second, a majority of children did not evidence an understanding of counting until 11 years of age. If a conception of the formal organization of counting were reducible to understanding the numerical significance of one-to-one correspondence relations, then children should achieve such a conception at about 5 years of age, the typical age norm for understanding the numerical significance of one-to-one correspondence (e.g., Piaget, 1952; Rothenberg, 1969). The late age norm of 11 years implies a formal understanding of counting is not reducible to simply an understanding of one-to-one correspondence relations, but rather, requires a further developmental process concerning the arbitrary character of conventional systems of reference.

Two additional hypothesis follow from the analysis that understanding the conventional nature of counting systems is dependent upon, but not reducible to understanding the numerical significance of one-to-one correspondence relations. First, children who have regular experience with alternative conventional systems of signification, such as bilinguals, should demonstrate a more advanced conception of the organization of counting systems than their same-aged monolingual peers who are matched in their understanding of one-to-one correspondence relations. Second, children's understanding of number conservation should develop partially independently of children's formal understanding of counting. As pointed out in the introduction, although both concepts require an understanding of static one-to-one correspondences, they have some important differences. Conservation, unlike counting, necessarily requires the child to understand the invariance of one-to-one correspondence relations over spatial transformations. A formal understanding of counting, unlike number conservation, necessarily requires an appreciation of the conventional nature of numerical symbols. As a consequence, depending upon children's particular histories, some children should achieve a formal understanding of counting prior to number conservation, whereas others should achieve an understanding of conservation prior to counting.

In order to test these hypotheses we interviewed bilingual and monolingual first grade children on the formal counting and number conservation tasks. The specific predictions were that (1) bilingual children would more frequently demonstrate an advanced performance on the counting task than their monolingual peers, and (2) bilinguals would tend to develop concepts of counting prior to number conservation whereas monolinguals would tend to develop concepts of conservation prior to counting

STUDY 2

Method

Subjects

The sample consisted of 32 six year old children composed of two groups, 16 monolingual and 16 bilingual children. The bilingual children spoke both French and English. Both groups of children were from middle class elementary schools in the New York area.

Procedure

Children were administered procedures to assess their understanding of number conservation (including static one-to-one correspondences) and their understanding of counting. Only set size 3 on the no violation and numeral repetition counting conditions was administered since no effect was found for set size and no difference in performance was found between the two violation conditions in the previous study. The order of administration of the counting and conservation tasks was counterbalanced, as was the order of administration of the two counting conditions.

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Number conservation condition. (1) One-to-one correspondence phase. The interviewer presented the child with a linear array of nine 2 cm diameter checkers and an available set of similar checkers. The interviewer told the child to put out one checker for each one in the model row. The child's reproduction was recorded. If the child did not make an accurate reproduction, the interviewer helped the child. Immediately following this task, the number conservation phase was administered. (2) Number conservation phase. Once the two arrays were aligned in one-to-one correspondence, as indicated in the previous phase, the interviewer said, "Now, do we both have the same number?" Once the child responded affirmatively, the interviewer said, "Now watch what I'm going to do." The model array was then spread apart and the child was asked, "Do we both have the same number, or does one of us have a bigger number?" "How do you know?"

The scoring system for the one-to-one correspondence and spatial transformation phase of the number conservation task each consisted of two levels, pass or fail. A "pass" score on the one-to-one correspondence phase consisted of producing an accurate copy of the model array. A "pass" score on the conservation task consisted of producing

conservation judgments for each spatial transformation and an appropriate conservation explanation.¹

Results

There were no differences between the bilingual and monolingual's levels of understanding of static one-to-one correspondences. Of the 16 bilinguals, 12 passed the static one-to-one correspondence assessment, and of the 16 monolinguals, 11 passed the one-to-one correspondence assessment procedures ($\chi^2=.15, p=n.s.$). Similarly, there were no differences in the bilingual and monolingual groups understanding of number conservation. Of the 16 bilinguals, 8 passed number conservation, and of the 16 monolinguals, 8 passed number conservation ($\chi^2_{(1)}=0, p=n.s.$). In contrast, an analysis of children's performances on the counting tasks revealed that, as predicted, a greater number of bilingual children than monolingual children achieved Level 2 performances for both of the counting tasks (no violation, $\chi^2_{(1)}=8.29, p < .01$; numeral repetition condition, $\chi^2_{(1)}=10.29, p < .01$).

Table 3 contains the performance profiles of the monolingual and bilingual children across both the no violation condition and conservation task, and Table 4 contains the same information for the numeral repetition condition and the conservation task. An analysis of the bilingual children's profiles revealed that they performed better on the counting than on the number conservation tasks, and this difference achieved statistical

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significance for one of the two counting conditions (sign tests, no violation/number conservation, $N=9$, $x=3$, $p < .25$; numeral repetition/number conservation, $N=6$, $x=0$, $p < .02$). In contrast, an analysis of the monolingual children's profiles revealed that they performed better on the number conservation task than the counting tasks and that this difference also achieved statistical significance for one of the two conditions (sign tests, number conservation/no violation, $N=8$, $x=1$, $p < .04$; number conservation/numeral repetition, $N=8$, $x=2$, $p < .15$). When the profiles across counting and number conservation tasks from the bilingual and monolingual populations are pooled, the expected independence between these two conceptual developments was revealed. That is, there was no difference in the frequencies in which children achieve an understanding of one of these concepts before the other (sign tests, no violation/number conservation, $N=17$, $x=7$, $p=n.s.$; numeral repetition/number conservation, $N=14$, $x=6$, $p=n.s.$

Discussion

The results of this study were consistent with both predictions. First, the bilingual children who were matched with the monolinguals in their levels of understanding of one-to-one correspondences and number conservation achieved more advanced performance on the counting tasks. This finding supports the interpretation that it is understanding the conventional properties of symbol systems, rather than understanding the numerical significance of one-to-one

correspondence relations that has delayed monolingual children's mastery of the counting task. Second, as expected, there was some tendency for monolinguals to achieve an understanding of number conservation prior to counting and bilinguals to achieve an understanding of counting prior to number conservation. Moreover, when monolingual and bilingual populations were pooled, there was no evidence that there is an invariant order in achieving a formal understanding of counting and number conservation. There was no difference in the number of children who achieved a higher level on counting than on conservation than who achieved a higher level on conservation than on counting.

In order to provide a further test for the formulation that children's formal understanding of counting is not reducible to simply an understanding of the numerical significance of one-to-one correspondence relations, an additional study was conducted. It was reasoned that if monolingual children have difficulty demonstrating a mature performance on the counting task due to their failure to understand the arbitrary character of numerical symbols, then manipulating the particular set of symbols used in the task may alter the sophistication of children's performances. Specifically, children who failed to demonstrate a mature understanding of counting when the alphabet was pitted against number names may be able to demonstrate a more sophisticated understanding of counting when the violation in the conventional feature is not so salient. If

the substitution set for the standard is not embedded in other functional contexts for the child (the alphabet is used for spelling, not counting), the monolingual children should demonstrate a considerably more sophisticated understanding of the organization of counting systems than previously demonstrated. In order to test this hypothesis, we interviewed an additional population of monolingual first graders. Half of the population received the original task format and half received a task condition in which one puppet counted using Chinese characters, and the other counted with Western numerals.

STUDY 3

Method

Subjects

The sample consisted of 32 6-year-old monolingual children who attended a middle class elementary school.

Procedure

One half of the sample received the same counting conditions described in Study 2 in which children were presented with the Bert-A and Ernie-1 puppets (the data obtained in Study 2 were used in this study). The other half of the children were administered the identical procedure, however, in place of letters Bert counted using Chinese numerals. A new Bert puppet with a Chinese numeral drawn on its back was used, and, prior to the task administration, children were taught to count to three using the Chinese numerals.

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Results and Discussion

An analysis of children's scores across the Chinese and alphabet forms of the task revealed that children more frequently achieved Level 2 performance on the Chinese form of the task for both counting conditions (no violation, $\chi^2_{(1)}=15.13$, $p<.001$; numeral repetition condition, $\chi^2_{(1)}=10.29$, $p<.01$). Thus, by using a set of numerals that was a less salient violation of the conventional aspect of numerical representation (Chinese numerals), monolingual children demonstrated a more sophisticated understanding of the organization of counting than was previously found in studies 1 and 2.

GENERAL DISCUSSION

Three general hypotheses were considered in the present research, each of which follow from the formulation presented in the introduction. The first hypothesis was that there should be general developmental changes in children's formal understanding of counting. The second was that these changes are not reducible to developmental changes in children's understanding of static one-to-one correspondence relations but, rather, require an additional understanding of conventional aspects of numerical reference. The third hypothesis was that there should be a partial independence in the development of a formal

understanding of counting and concepts of number conservation. In order to provide data which would bear on these hypotheses, a series of three studies was conducted. In Study 1, children across a wide age range were interviewed on the Bert-A/Ernie-1 tasks. In Studies 2 and 3, new populations of 6-year-old children were assessed on the same type of counting task, but two variables were manipulated. In Study 2, subject populations who had different histories of experience with conventional systems of reference (bilingual/monolingual comparisons) were administered the Bert-A and Ernie-1 tasks. In Study 3, tasks which employed numeral lists that had different degrees of "functional embeddedness" in activities other than counting (Chinese numerals/alphabet numerals) were administered to monolingual children.

The findings of the present studies supported the three general hypotheses. First, children's understanding of the formal organization of counting systems undergoes a qualitative change over development. Young children do not understand that any ordered list of elements can be used as a counting system. They typically judged that Bert-A was a poor counter even though he established one-to-one correspondence relations between letters and objects. Over the course of development, children achieve a formal understanding of counting. By 11 years of age children judged that Bert-A's counting was adequate.

Second, several sources of evidence indicate that a formal understanding of counting is not reducible to an understanding of static one-to-one correspondence relations, but rather, requires a further conceptual

development concerning the arbitrary character of conventional reference. For instance, children demonstrated an understanding of the numerical significance of one-to-one correspondence relations in the violation conditions by judging that Ernie-1 was a poor counter. At the same time, many of these children also judged that Bert-A was also a poor counter, and thereby demonstrated that they did not understand that any ordered set of conventions could substitute as a set of numerals. Further support for the irreducibility of understanding of conventions to an understanding of one-to-one correspondences was produced in the comparisons between the bilinguals and monolinguals. Children who regularly used alternative systems of signification (bilinguals) showed a superior performance to children who did not have such experience (monolinguals) but who had achieved the same level of understanding of one-to-one correspondence relations. Reciprocally, when the counting conditions were presented in a task context in which the alternative set of numerals was not embedded in other functional contexts for monolinguals (Chinese numerals), monolinguals' performances on the counting conditions significantly improved.

Third, the findings demonstrated, as predicted, the partial independence between the developments of a formal understanding of counting and number conservation. On the present formulation, to achieve a formal understanding of the organization of counting requires knowledge about the arbitrary character of numerical reference. That is, the child

must understand that the numeral is not a number itself, but is an arbitrary symbol that stands for or refers to number. In contrast, number conservation does not require an understanding of the arbitrary character of numerical reference. However, unlike counting, it necessarily requires an understanding of the invariance of one-to-one correspondence relations over spatial transformations. On the present formulation, these different conceptual difficulties should lead to a partial independence of these conceptual developments, and this is precisely what we found. Bilinguals, a group who have direct experience with alternative systems of signification, tend to develop an understanding of counting prior to number conservation, whereas the monolinguals, a group without such experience, tended to develop an understanding of conservation prior to a formal understanding of counting.

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Footnotes

¹Conservation scores were also assessed using a "judgments only" criterion. All children who judged conservation across each of the two transformations, also gave adequate explanations.

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Table 1

Age Trends in Performance across the No Violation Condition

Age Group	Set size 3		Set size 6	
	Level 1	Level 2	Level 1	Level 2
4-year-olds	14	2	11	4
8-year-olds	12	4	11	5
11-year-olds	3	13	3	13

Table 2

Age Trends in Performance Across Numeral Repetition and Numeral Omission Conditions

Age Groups	Numeral Repetition						Numeral Omission					
	Set size 3			Set size 6			Set size 3			Set size 6		
	Level	Transi-	Level	Level	Transi-	Level	Level	Transi-	Level	Level	Transi-	Level
	1	tional	2	1	tional	2	1	tional	2	1	tional	2
4-year-olds	4	10	4	5	7	5	3	11	4	4	8	3
8-year olds	0	7	9	0	7	9	0	7	9	0	7	9
11-year olds	0	3	13	0	3	13	0	3	13	0	3	13

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Table 3

Frequency Distribution of Monolingual and Bilingual Children's
Pass/Fail Profiles Across the No Violation Condition and the
Number Conservation Task

Across Task Profiles ^a				
Language Group	No Vio/ <u>Cons</u>	<u>No Vio</u> /Cons	No Vio/ <u>Cons</u>	No Vio/Cons
Bilinguals	2	3	6	5
Monolinguals	7	7	1	1
Bilinguals and Monolinguals	9	10	7	6

^aThe abbreviations refer to the following counting/conservation profiles.
NoVio/Cons, fail scores on both the no violation condition and conserva-
tion; NoVio/Cons, fail score on the no violation condition but pass
score on the conservation condition; NoVio/Cons, pass score on the no
violation condition but fail score on the conservation condition;
NoVio/Cons, pass scores on both the no violation condition and conser-
vation.

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Table 4

Frequency Distribution of Monolingual and Bilingual Children's
Pass/Fail Profiles Across the Numeral Repetition Condition
and the Number Conservation Task

Across Task Profiles ^a				
Language Group	<u>Num R/Cons</u>	<u>Num R/Cons</u>	<u>Num R/Cons</u>	Num R/Cons
Bilinguals	2	0	6	8
Monolinguals	6	6	2	2
Bilinguals and Monolinguals	8	6	8	10

^aThe abbreviations refer to the following counting/conservation profiles.
NumR/Cons, fail scores on both the numeral repetition condition and conservation; NumR/Cons, fail score on the numeral repetition condition but pass score on the conservation condition; NumR/Cons, pass score on the numeral repetition condition but fail score on the conservation condition; NumR/Cons, pass score on both the numeral repetition condition and conservation.

Section IV.

Children's Use of Counting to Produce Numerical Information

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Gesture in Early Counting: A Developmental Analysis

Geoffrey B. Saxe and Rochelle Kaplan

The Graduate Center of the City University of New York

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Running head: Gesture in Early Counting

Gesture in Early Counting

Abstract

This study examines the changing function of pointing gestures in children's counting activities. Children at 3 age levels (2, 4, and 6 years) were required to count objects in two conditions, one in which they could not use pointing gestures and the other in which they were encouraged to do so. The results indicated that with increasing age, children's counting accuracy improved across both conditions but that it was only at the 4 year age level that children benefitted by their use of gesture on the counting tasks. The findings are discussed with respect to existing models of number development.

Gesture in Early Counting: A Developmental Analysis

The activity of counting involves establishing a one-to-one correspondence between two sets of elements. One set, the "standard," is a conventionally defined list of symbols for number which is always used in counting (e.g., "one," "two," "three,"...). The second set, the "variable," is a target array of elements which varies from one count to another. The subject's aim in counting is to establish a one-to-one correspondence between the two sets such that the standard serves to notate a progressive summation of correspondences with the variable set.

Pointing gestures are one vehicle that an individual employs to help insure that an accurate one-to-one correspondence between the standard and variable sets is achieved. Although various researchers have documented the pervasiveness of pointing gestures in 2- and 3-year olds' counting activity (e.g., Gelman & Gallistel, 1978; Saxe, 1977), whether young children are actually attempting to use their pointing gestures to serve a numerical function is a matter of some controversy. The purpose of this paper is to contrast two models of number development and the function which these models attribute to children's pointing gestures in their counting activities.

Gelman and Gallistel (1978) have proposed a model in which they argue that very young children know basic principles underlying the counting system, such as the necessity to assign the standard set in one-to-one correspondence with a variable set. They claim that pointing gestures are a means by which children effect an accurate correspondence. As support for this formulation, Gelman and Gallistel report that young children tend to achieve a high degree of accuracy with small sets of objects; however, they also note that as set

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size increases, counting accuracy declines. They argue that for larger set sizes, extra-logical features (e.g., motor incoordination) interfere with the child's accurate application of the standard set to the variable set. Over development, the motor coordination improves and therefore counting accuracy for larger sets improves as well. Thus on the Gelman and Gallistel formulation, the function of gesture does not change in children's counting activities, rather, with increasing age, gesture is simply used with greater and greater degrees of efficiency.

A contrasting view is that the function of gesture in children's counting activities does change over development. Unlike Gelman and Gallistel, various researchers (e.g., Piaget, 1952; Inhelder & Piaget, 1964; Saxe, 1979) have argued that when very young children enumerate elements, they do not consider each element as a member of a larger set of elements. As a consequence, the correspondences young children produce between standard and variable sets are not limited only to one-to-one correspondences since children do not use their counting to serve a numerical function.

In this paper, we propose a model about the function of gesture in counting which is an extension of this second view of number development. On the proposed model, there are a sequence of three phases which characterize the changing function of gesture in children's counting activity.

During the first phase, the young child uses gesture in counting, but does not use gesture to serve a numerical function. Rather, children use pointing gestures in their enumerative activities in a manner similar to pointing gestures in their early word use. Just as young children use

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pointing gestures to assign names to objects, they use pointing gestures to assign a succession of numerals to objects and do not feel the necessity to point to all of the elements in a set or to point to elements only once.

During this phase children's use of gestures may even inhibit their counting accuracy. When set size is small (e.g., 2 or 3 elements), children may use the configuration of the whole set to help define the number of discrete elements, and not need to count the set, element by element, to achieve an accurate numerical representation (cf. Klahr and Wallace, 1976; Saxe, 1979). As a consequence it is plausible that if children use gestures to count small sets they may break-up the configurational aspects of an array, and this may interfere with an accurate numerical representation.

During a second phase, as children achieve an understanding that each element which is enumerated can be considered as a member of a summation of elements, children should attempt to apply one and only one number name to the elements of a set; however, they may have considerable difficulty establishing these correspondences with eye movements and, therefore, they should use their pointing gestures to help them accomplish this application. Thus, as compared to the first phase, children's counting accuracy should generally improve, and this improvement should be facilitated by their use of gesture in counting.

Finally, during a third phase, children may become more capable of tracking elements with eye movements (Bruner, 1966) and therefore their use of gesture may exert little influence on their counting accuracy in simple tasks. During this phase children may need to draw upon their use of gesture only in

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situations in which they are not confident in their ability to produce an accurate count.

In order to evaluate the relative adequacy of these two models, a novel task procedure was devised. Children at three age levels were asked to count elements at various levels of set size under two conditions, one in which they were allowed to use gestures and the other in which they were not. If gesture does not change in function over development as Gelman and Gallistel propose, then one should expect that children should benefit by their use of gestures in their counting to some degree at each age level. However, if children's use of pointing gestures do change in function over development, then one should observe a certain pattern of differences in children's performance in these two conditions at different age levels. Specifically, at 2 years of age, children should not use number names to represent numerical relations. Therefore, gesture would not serve a numerical function and children's performances should not differ in the gesture allowed and gesture inhibited conditions. To the extent that differences are expected, children should actually perform more poorly on the gesture allowed conditions for small set sizes since the act of pointing in the enumeration of the elements may break up the configuration of the elements. At four years of age, children should begin to use number names to represent numerical relations and therefore children's use of gesture should begin to serve a numerical function; consequently, children's counting accuracy should generally improve, and this improvement should be manifest to a greater degree in the gesture allowed condition than in the gesture inhibited condition. At six years of age,

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children's use of gesture should be unnecessary to produce an accurate count in simple counting tasks. Therefore, there should be no difference in children's counting accuracy across the two conditions.

MethodSubjects

The subjects included 42 children, 14 at each of three age levels (2, 4, 6 years), from private schools in New York City and the surrounding suburbs. The 2-year-old group included 9 boys and 5 girls (mean age = 2 years, 9 months, SD = 2.50 months). The 4-year-old group included 5 boys and 9 girls (mean age = 4 years, 0 months, SD = 3.20 months). The 6 year old group included 5 boys and 9 girls (mean age = 6 years, 2 months, SD = 3.40 months).

Materials

The materials included a miniature Mickey Mouse doll, 13 homogeneous linear arrays consisting of evenly spaced animal pictures pasted on 27 cm x 25 cm cardboard rectangles, and a 29 cm x 45 cm x 23 cm black and white striped box.

The animal displays consisted of three each of 2, 3, 7, and 8 item arrays. An additional two-item array was used as a training item for orienting the child to the task.

The box was used to represent a cage for the animals and was provided with a 13 cm x 15 cm opening in the front through which the child could view the animals and a 27 cm slit across the top in which the animal displays could be placed one at a time. In addition, the box had a 9 cm x 25 cm opening across the lid which allowed light to shine in on the displays.

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Procedure

Children were presented with the series of 12 linear arrays under two conditions. In the Gesture Allowed condition, the arrays were placed on the table directly in front of children and they were encouraged to point to the objects as they counted. In the Gesture Inhibited condition, children were presented with the same arrays placed upright inside the "cage." The objects were only visible through the opening in the front of the cage making it impossible for them to point to the objects as they counted. The order of the Gesture Allowed and the Gesture Inhibited conditions was counterbalanced across subjects within age groups as was the first trial presentation for large or small array size. The order of large and small array presentations was alternated within each condition for all subjects. An initial pretest trial consisting of a two object array, however, was introduced to all subjects under both conditions in order to establish a counting and pointing or a counting and no pointing set for responding.

All children were interviewed individually within the classroom setting or in a nearby familiar room. At each interview the child was introduced to a small Mickey Mouse doll and told the following story: "Mickey Mouse works in a zoo some of the time and last night all of his animals ran away. This morning he has gone all over the city (town) in order to find them and now he thinks he has them all back. He wants to be sure he has all of them and so he has to count them. Will you help him count his animals?"

Once the child has agreed to help Mickey, he was told: "Before you count the animals, can you just count out loud for Mickey so he can hear you

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count?" If the child did not seem to understand, he was told, "Just say the numbers like you were counting." Further encouragement was given by the examiner saying, "Go ahead, start with 1, 2, 3..." Only those children who could generate at least 10 numeral names in a row, either using the conventional number list or their own idiosyncratic one, were included in the sample. All unconventional lists were recorded at the beginning of each protocol. In addition, all responses of the younger children were tape recorded. Any child who continued rote counting past 20 was told "That's just fine. You can stop now."

Following this, the child was presented with one of the two counting conditions.

Gesture Inhibited condition. The child was told the following: "Sometimes Mickey likes to keep his animals indoors in a cage like this one—especially in the winter when it is cold. You look inside the cage here and tell Mickey how many rabbits he has. The 2 object rabbit pretest array was then presented. If the child attempted to place his hand in the cage, he was told: "No, you must never put your hands inside an animal's cage. Just keep your hands on the top of the box like this (the examiner helped place the child's hands on top)." Most 2-year olds were given this latter instruction as a matter of course before they began to count. On all subsequent trials in which the child attempted to point, the instruction was repeated.

Gesture Allowed condition. The child was told the following: "Sometimes, especially when the weather gets warm, Mickey likes to keep his animals out in an open field (examiner added "like at the Bronx Zoo" for older New York City children) like this." The two object rabbit pretest array was then

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presented. "How many rabbits does Mickey have?" If children did not point as they counted, they were told, "You can point to the animals to help you count."¹

In either condition, if children did not count spontaneously, but offered a single number response, they were told, "Yes, that's fine. Now can you count the rabbits?" This cue was repeated on any trial in which counting did not occur spontaneously.²

The procedure continued until children had made 24 counts. At that point the examiner praised them and thanked them for Mickey, terminating the interview.

Results and Discussion

The series of number names children used on each counting trial was recorded. In order to determine a counting accuracy score for each child, the difference between the number of number names and the actual set size of the trial array was determined. An overall error score for each set size was then calculated for each child by summing the absolute values of these differences across the three trials for each of the two conditions (Gesture Inhibited and Gesture Allowed).

Figure 1 contains a plot of median overall error scores over the three age levels for each condition. As expected, with increasing age, Kruskal-Wallis one-way ANOVAs revealed that children committed fewer errors at each level of set size on both the Gesture Allowed condition (set size 2, $\chi^2_{(2)} = 31.6$, $p < .001$; set size 3, $\chi^2_{(2)} = 23.4$, $p < .001$; set size 7, $\chi^2_{(2)} = 31.05$, $p < .001$; set size 8, $\chi^2_{(2)} = 31.5$, $p < .001$) and Gesture Inhibited condition (set size

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2, $\chi^2_{(2)} = 23.2$, $p < .001$; set size 3, $\chi^2_{(2)} = 21.3$, $p < .001$, set size 7, $\chi^2_{(2)} = 21.3$, $p < .001$; set size 7, $\chi^2_{(2)} = 28.6$, $p < .001$; set size 8, $\chi^2_{(2)} = 32.4$, $p < .001$). Kolmogorov-Smirnov two sample tests were used to determine whether there were differences in children's error scores across the two conditions at each age level. These analyses revealed that at 2 years of age, there were no significant differences across the conditions at any level of set size, however, as predicted, there was some tendency for children at set size 2 to produce fewer errors in the Gesture Inhibited condition as opposed to the Gesture Allowed condition ($N = 14$, $d = 4$). Thus, if gesture is implicated at all in counting accuracy at 2 years of age, it appears to be an interfering as opposed to a facilitative factor. At 4 years of age, children's counting was more accurate in the Gesture Allowed than in the Gesture Inhibited conditions for set sizes 7 ($N = 14$, $d = 9$, $p < .01$) and 8 ($N = 14$, $d = 8$, $p < .01$). For set sizes 2 and 3, children's performances approached a ceiling, although there was a tendency for children to perform better on the Gesture Allowed than the Gesture Inhibited condition (set size 2, $N = 14$, $d = 2$; set size 3, $N = 14$, $d = 4$). Thus, as counting accuracy improves from 2 to 4 years of age, pointing gestures begin to play a facilitative role in children's counting. At 6 years of age, children's performance reached a ceiling across all set size conditions; therefore, there was no difference revealed in children's performances across conditions. It is of some interest to note, however, that many of the 6-year-olds chose not to gesture spontaneously in the Gesture Allowed conditions. Particularly, on the small set sizes, these children needed to

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be prompted in order to use pointing gestures. Even with prompting sometimes these children tracked the elements without gesture in both conditions and produced accurate counts.

Insert Figure 1 about here

In the present study we documented the pervasiveness of children's pointing gestures in their counting activities. It is of interest to note that not only did the younger children use pointing spontaneously in the Gesture Allowed condition, they often tried to point through the window in the box in the Gesture Inhibited condition. Despite the use of gesture in counting at all age levels, the present findings support the thesis that the function of pointing gestures changes in children's counting activity over the course of development.

Consistent with the view that children's recitation of number names in enumerative activities shifts from a non-numerical to a numerical function with increasing age, our findings support the thesis that the function of children's pointing gestures changes in their counting. On this view, although very young children (2-year-olds) use gestures while counting, these gestures are used as in other naming activities. At this point in development gesture is employed to reference a word to an object and does not serve a numerical function. As children's counting begins to take on quantitative significance, children attempt to establish one-to-one correspondence relations between numerals and objects. With this change, the use of gesture which was

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initially created by the child to serve a referential function is gradually transformed in function to serve a new purpose—to help count accurately.

Thus, gesture at 4 becomes in part the referential act that it was at 2 and at the same time the external support that is deployed in counting when needed at more mature developmental levels. Finally, by 6 years of age children's use of gesture is a differentiated feature of the counting process. Children only need to deploy gesture in situations in which they doubt their ability to achieve an accurate count without the external support of gesture.

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Footnotes

¹Some of the six year olds did not use overt pointing gestures despite the encouragement to do so, but instead tended to nod their heads or just simply shift their gazes as they counted. When such responding was consistently accurate, the child was no longer repeatedly told to point. Rather it was assumed that the pointing gesture had given way to more subtle tracking gestures.

²Some of the six year olds continued to offer a single accurate number name for small array sizes despite instructions to count on the earlier presentations. After 3 such accurate responses, these children were no longer instructed to count small array sizes.

Configurations as Figurative Supports
In Young Children's Counting

Stephen Sicilian and Geoffrey B. Saxe

The Graduate Center of the City University of New York

Running head: Configurations as Figurative Supports

Configurations as Figurative Supports

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Abstract

The purpose of this study was to determine whether the spatial configuration of a set of elements plays a facilitative role in children's numerical evaluations. To this end, 3-year-olds were asked to evaluate the cardinal number of small sets (2 or 3 elements). The elements were presented either successively or simultaneously and children were required to make their evaluations verbally (stating the number name) or non-verbally (putting out the same number of objects). Results indicated that children are more likely to produce accurate cardinal judgments when provided with simultaneous as opposed to successive object presentations. Variations in children's response modality revealed no difference in their ability to produce accurate cardinal judgments. The results support the thesis that the spatial configurations of small sets provide a figurative support for children's emerging counting abilities.

Configurations as Figurative Supports

In Young Children's Counting

Two divergent models have been proposed to account for the finding that very young children are able to demonstrate numerical concepts when small set sizes are used in number tasks but not when larger sets are used. Klahr & Wallace (1976) argue that young children produce accurate cardinal judgments when evaluating small sets by identifying the configurational arrangement of small sets through some direct apprehension mechanism ("subitizing"); thus, children learn to label those configurations with numerals (e.g., "two" or "three") much the same as they use words to label any other physical object (e.g., tree, house, dog) without the ability to actually quantify sets by counting. Alternatively, Gelman & Gallistel (1978) have argued that young children understand basic principles of counting before they can subitize. They argue that an inherent difference between small and large set size tasks is that large sets require extranumerical skills (e.g., attentional and perceptual abilities) which the young child does not possess. Thus, on the Gelman & Gallistel model, difficulty with these extranumerical skills rather than a lack of understanding of fundamental counting principles is responsible for the child's poor performance.

The purpose of the present study was to evaluate the relative adequacy of these models of how children quantify small sets. Children were presented with a task in which they were required to evaluate the cardinal number of small sets under two presentation conditions. In one condition, children were

presented with elements successively eliminating the possibility of subitizing. In the second condition the elements were presented simultaneously providing a spatial configuration which could be subitized. On the Klahr & Wallace formulation, children should produce more accurate evaluations in the simultaneous than in the successive presentation condition since they should be able to use the configuration as a cue for an accurate cardinal response. On the Gelman & Gallistel formulation, there should be no difference in children's performance across these task conditions in that children should count on each condition to produce an accurate response. In order to provide more confidence in the outcome of the findings, children were required to produce a cardinal judgment in one of two response modes, either a verbal numeral or a physical tally of the target set.

Method

Subjects

The sample consisted of 40 3-year-olds (22 females and 18 males, mean age = 3 years, 9 months, standard deviation = 3 months). All children attended nursery schools in middle class suburban areas of New York City.

Materials

The materials consisted of a 7 cm x 16 cm drawing of a young girl fixed to a 38 cm x 50 cm sheet of matboard. A 3 cm x 6 cm window was cut in the matboard. The window was located below the girl's extended hand and above a 5 cm x 5 cm drawing of a bucket. A rotating matboard disk was attached behind the large sheet so that a rotation caused a series of 1.5 cm pictures of red balls to pass by the window. A rotation of the disc thus caused the girl to appear to be throwing balls in the bucket.

Procedure

Children were administered six trials of the successive presentation condition (three at set size 2, and three at set size 3) and six trials of the simultaneous presentation condition (three at set size 2, and three at set size 3) in one of two response modes (verbal or non-verbal). The order of presentation on the conditions was counterbalanced and children's assignment to the response modes was randomized.

Successive condition. A set of red balls appeared in the window one at a time, and each ball was visible for one second. The children in the verbal response group were told "This little girl's name is Claire and she is going to throw some red balls into this bucket. I want you to watch carefully and tell me how many balls she throws into the bucket." Children in the non-verbal group were supplied with five 2 cm diameter red balls contained in a small plastic dish. They were told "This little girl's name is Claire and she is going to throw some red balls into this bucket. I want you to watch carefully and hand me just as many balls as she throws into the bucket."

Simultaneous condition. Sets of either two or three red balls appeared in the window simultaneously. The three ball display was exposed for three seconds, and the two ball display was exposed for two seconds.

Results and Discussion

Children were assigned a score of either 0 (incorrect) or 1 (correct) for each trial. Children achieved more accurate cardinal responses on the simultaneous than on the successive presentations (Wilcoxon test for large N , $Z = 3.31$, $p < .001$) and this effect was manifested in both the verbal condition

(Wilcoxon test for small n , $N = 13$, $T = 5$, $p < .01$) and non-verbal condition (Wilcoxon test for small n , $N = 16$, $T = 29$, $p < .05$). Children did not significantly differ in their performance on the two response modes, although there was a tendency for children to perform better in the verbal than the non-verbal response mode (Kolmogorov-Smirnov two sample test, $N = 20$, $K_D = 3$, $p < .1$).

The finding that the children in the present study produced more accurate cardinal judgments in the simultaneous as opposed to the successive conditions tends to support Klahr & Wallace's as opposed to Gelman & Gallistel's formulation. However, it is significant to note that 80% of the children correctly identified the arrays on at least 3 of their 6 attempts during the successive presentations. Thus, although children produced more accurate cardinal judgments in the simultaneous condition, there is evidence, nonetheless, that young children used counting to quantify sets. In light of this finding an alternate explanation was formulated. Saxe (1979) suggests that when young children enumerate sets, they tend either to consider the elements only as discrete individuals or as a conglomerate and that they have considerable difficulty coordinating these two perspectives into a progressive summation of elements, the cognitive operation involved in counting. Consistent with the Saxe formulation, the findings of the present study suggest that the configuration of a small set may aid the child in understanding a progressive summation. In particular, for small sets, the configuration of the whole may support the child's numerical representation of the discrete number of elements. Thus, children may use the configuration of small sets as a figurative support for their developing ability to produce numerical evaluations by counting.

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Children's Interpretations of Their Counting Accuracy:

A Developmental Analysis

Geoffrey B. Saxe and Stephen Sicilian

The Graduate Center of the City University of New York

Running head: Children's Interpretations

Children's Interpretations

Abstract

This study examines differences between 5-, 7-, and 9-year-olds' ability to estimate their counting accuracy on tasks of three levels of counting difficulty. Results indicated that younger children tended to believe that they counted accurately independent of their actual accuracy. With increasing age children's estimates of their counting accuracy increasingly corresponded to their actual counting accuracy. These findings are interpreted with respect to children's developing understanding of counting as an empirical operation.

Children's Interpretations of Their Counting Accuracy:

A Developmental Analysis

Researchers interested in the development of "metacognition"—the ability to monitor one's own cognitive processes and activities—have analyzed the way in which children approach and solve various types of memory tasks (cf. Brown, 1978; Flavell, 1979; Flavell & Wellman, 1977). This research has demonstrated that with increasing age, children are better able to reflect about their own memory limitations and to use effective mnemonic strategies. In the present study, the concern for understanding metacognitive processes is extended to the area of number representation in order to reveal some basic developmental changes in children's use of counting as an empirical operation. The particular focus is children's changing abilities to reflect about their own counting accuracy.

In counting, a subject establishes a succession of one-to-one correspondence relations between verbal numerals and elements of a target set and uses these numerals to notate the progressive summation of these correspondence relations. On the basis of a count, a subject makes a judgment of the number of elements in the target. Since the activity of counting is an empirical operation—a means of producing factual information about sets—a subject's numerical judgment based upon counting is not true by logical necessity (as, for instance, are judgments about number conservation). Rather, the truth value of a judgment based upon counting is dependent upon the precision with which the empirical operation is performed on a set. As a consequence, a subject should have more or less confidence in the truth value of a particular

count depending upon the subject's perception of the precision with which correspondences were established between numerals and the target elements.

In the present study, children were asked to count arrays which differed in their spatial arrangements such that for some arrays it would be extremely difficult for children to produce an accurate count whereas for other arrays it would be relatively easy. Based upon the literature on metacognition, we suspected that young children who were at an age level at which they understood the logic of counting, would nonetheless not understand that they could not be certain of their counting accuracy. These children therefore would not necessarily assume that under one condition they would be any more or less accurate in a count than on another condition. Over the course of development, children should begin to reflect upon the process of counting. As a consequence, children should have a greater understanding that in certain contexts they could be more certain of their accuracy than in other contexts.

Method

Subjects

The sample consisted of 15 children at each of three age levels (five, seven, and nine). The 5-year-old group consisted of 9 females and 6 males

with an average age of 5 years, 7 months ($SD = 2.5$ months). The 7-year-old group consisted of 8 females and 7 males with an average age of 7 years, 6 months ($SD = 3$ months). The 9-year-old group consisted of 7 females and 8 males with an average age of 9 years, 8 month ($SD = 2$ months). All subjects were drawn from a single elementary school in a middle class suburb of New York City.

Procedure

Each child was presented with three types of counting tasks (Beaker, Fixed Array, Loose Array) which were designed to be ordered in their difficulty levels. Each of the three tasks was administered in three trials and on each of these trials the level of set size was varied slightly (28, 29, or 30 elements). The order of presentation of the three tasks was counter-balanced. Prior to the administration of the tasks children were asked to count to 40 and, if they did not reach this criterion they were not included in the study.

The tasks were administered to children individually. The interviewer and child sat on opposite sides of a table and the child was told: "I want to play some counting games with you. In some of the games blocks or beans will be loose and you will be able to move them if you want. In other games they will be either stuck on a piece of cardboard or placed in a jar and you will not be able to move them." Children were shown an example of each condition during this explanation.

At this point one of the trials described below was presented, and children were told: "Count and tell me how many beans (or blocks) there are here. After children responded and the number was recorded, the interviewer asked, "Are you

sure there are (child's response), or do you think you might be wrong?" If children responded that they were sure they were right, the next trial was administered in the same fashion. If children responded that they thought they might be wrong, the interviewer asked children, "What is the biggest number there could be?" They were then asked, "What is the smallest number there could be?" The answer to each of these questions was recorded. The same procedure was followed for each trial of all three tasks.

Beaker task. The materials for this task consisted of two beakers of blocks and one beaker of beans. In each beaker, some of the elements were concealed underneath and between others. Since the elements were contained in the beaker, the child could not touch or move the elements.

Fixed objects task. Two sets of blocks and one set of beans of the same dimensions used in the Beaker task were fixed to a piece of matboard in a random array. In this condition the child could touch each element separately; however, the elements could not be moved.

Loose objects task. Two sets of blocks and one set of beans of the same dimensions used in the Beaker task were presented in a loose pile. The child was able to manipulate elements freely.

Results

The first step of the analyses was to determine whether, in fact, our counting tasks led children to count the sets at different levels of accuracy, and whether these levels of accuracy were related to age. This information would allow us to proceed to the second step, to determine whether or not

differences in children's confidence in their counting accuracy were appropriately related to the difficulty of the conditions.

Counting accuracy Children's counting accuracy scores were determined by summing the absolute differences between their count and the actual number in the target set for each of the three trials within each condition. Figure 1 contains children's median error scores for each condition at each age level. A 3 x 3 ANOVA was performed using task condition (repeated measure) and age group as independent measures and error scores as the dependent measure. This analysis revealed significant effects for both of these factors (task condition, $F(2,84) = 34.6$, $p < .001$; age group, $F(2,42) = 5.61$, $p < .01$) and no significant interaction. Newman-Keuls post hoc comparisons revealed that, as expected, children were more likely to error on the beaker than on fixed array conditions ($p < .05$) and more likely to error on the fixed as opposed to the loose array condition ($p < .05$). Post hoc analyses also revealed that 5-year-olds errored to a greater extent than 7-year-olds ($p < .05$) and that 7-year-olds errored to a greater extent than the 9-year-olds ($p < .05$). In sum, these analyses conform to the a priori assumptions about the relative order of difficulty of the counting tasks.

Insert Figure 1 about here

Counting confidence. Children's "uncertainty" in their counting accuracy was operationally defined as the difference between their biggest and smallest estimates of the set size that they had just counted. Uncertainty scores were

determined by summing the absolute differences between children's biggest and smallest estimates of a target set for each of the three trials within a task condition. Figure 2 contains a plot of the median uncertainty scores for each condition over age group. A Kruskal-Wallis one way ANOVA revealed, as expected, that on each of the three conditions, children's uncertainty in their counting accuracy differed over age level (Beaker condition: $\chi^2_{(2)} = 18.6$, $p < .001$; Fixed Array condition: $\chi^2_{(2)} = 8.23$, $p < .02$; Loose Array condition: $\chi^2_{(2)} = 8.17$, $p < .02$).¹ Mann-Whitney U tests indicated that on each of the tasks there were significant differences between the 5-year-olds and the 7-year-olds (Beaker condition, $Z = 3.99$, $p < .0001$; Fixed Array condition, $Z = 2.46$, $p < .01$; Loose Array condition, $Z = 2.43$, $p < .01$), but no significant differences between the 7- and 9-year-olds.

Insert Figure 2 about here

In order to determine children's relative confidence in their counting at each age level, Wilcoxon tests were used. As expected, the 5-year-olds showed no differences in their levels of uncertainty confidence over the task conditions. In contrast, both the 7- and 9-year-olds showed appropriate differences in their levels of uncertainty over the conditions. The 7-year-olds were more uncertain in their counting in the Beaker condition than in the Loose Array condition ($Z = 3.06$, $p < .002$), and more uncertain in the Fixed Array condition than in the Loose Array condition ($Z = 2.82$, $p < .005$); however, the 7-year-olds showed no differences in their levels of uncertainty

between the Beaker condition and the Fixed Array condition ($Z = .07$, $p < .94$). Similar to the 7-year-olds, the 9-year-olds were more uncertain in the Beaker than in the Loose Array condition ($Z = 2.70$, $p < .01$). In contrast to the 7-year-olds, however, the 9-year-olds were appropriately more uncertain in the Beaker condition than in the Fixed Array condition ($Z = 2.35$, $p < .02$) and there was some tendency for them to be more uncertain in the Fixed Array than in the Loose Array condition ($Z = 1.6$, $p < .11$).

Discussion

The findings of the present study are consistent with the previous literature on metacognition which has indicated that, with increasing age, children more effectively monitor their own cognitive activities. On the tasks used in the present study, younger children did not "monitor" their counting such that they were aware they might have counted inaccurately in the different conditions. Thus, at 5 years of age, children were generally confident that they had achieved an accurate count regardless of the difficulty of the counting condition. At 7 years of age, children were less confident in their counting accuracy in the Fixed Array and the Beaker conditions, and appropriately more confident in the Loose condition. It was not until 9 years of age, however, that children's confidence levels reflected the actual difficulty levels of the counting tasks; at this age level, children were regularly less confident in the Beaker than in the Fixed or Loose Array conditions.

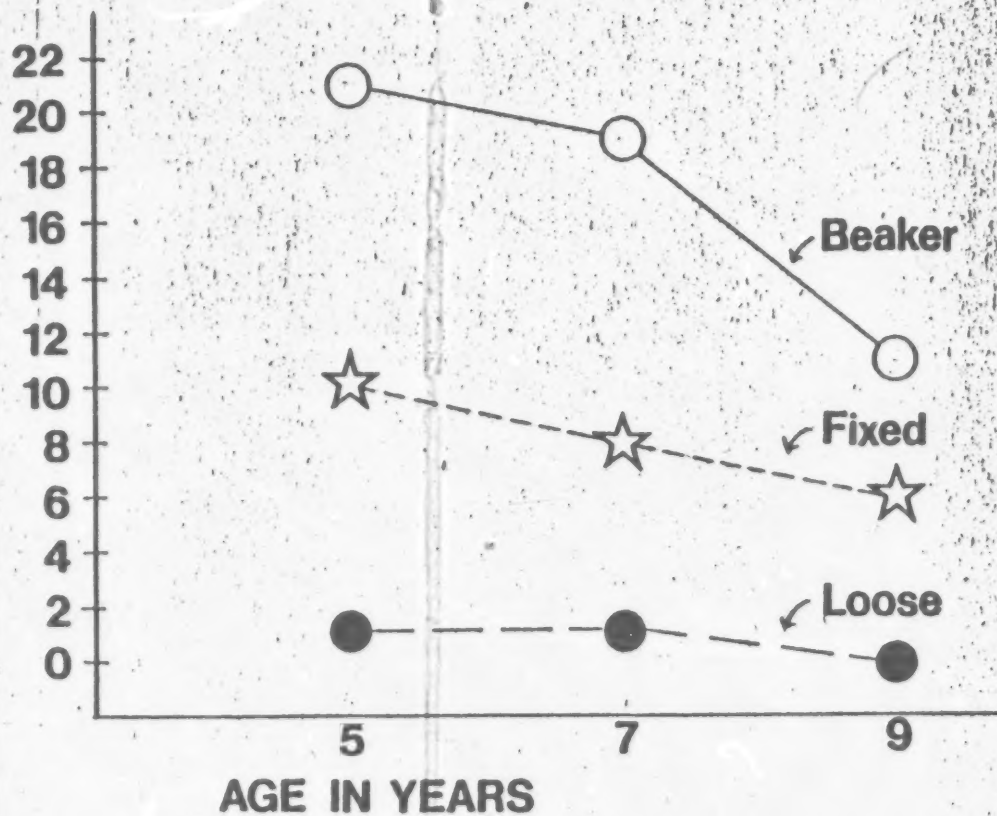
Although the present findings are certainly consistent with the research literature on metacognition, the findings are also informative about a developmental process which is an inherent aspect of numerical cognition itself, but

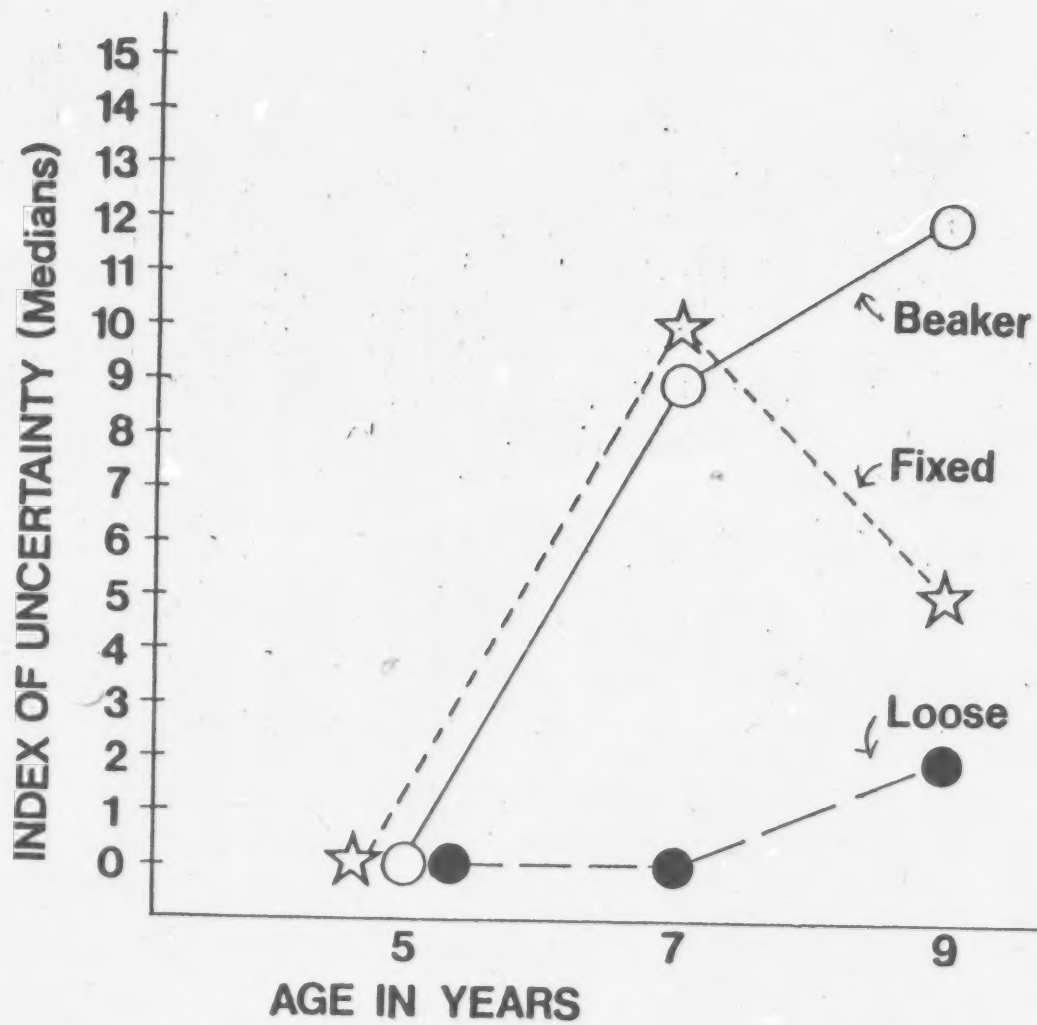
but largely neglected in the research literature. Unlike certain numerical concepts which are true by virtue of their logical necessity (e.g., number conservation), the truth value of numerical statements which are a product of counting are indeterminate. There is always the possibility that one may error in a particular count. Thus, in order to make effective use of information produced through counting a subject must make an interpretation of the accuracy of the count. The cognitive basis of the child's interpretation of counting accuracy constitutes a form of "empirical" or "experimental reasoning" which is an important complement of the deductive reasoning typically assessed on Piagetian tasks. The present study succeeded in documenting that this form of reasoning exhibits a developmental function. It remains a further research problem to explore the factors which influence the development of this form of experimental reasoning and the way in which the child uses empirical information produced through counting in forming arithmetical and physical concepts.

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INDEX OF COUNTING ERRORS (Medians)





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4. Children's use of counting to produce numerical comparisons and reproductions: The influence of set size and spatial configuration of sets.

To be submitted at a later date

Coding System for Children's Counting Behavior

1. Introduction

The activity of counting involves establishing a one-to-one correspondence between two sets of elements. One set, the "standard" is a conventionally defined list of symbols for number which is always used in counting (e.g., "one," "two," "three,"...). The second set, the "variable," is a target array of elements which varies from one count to another. The subject's aim in counting is to establish a one-to-one correspondence between the two sets such that the standard serves to notate a progressive summation of correspondences with the variable set.

The purpose of this report is to describe the procedures used to elicit counting behavior in young children as well as a notational system used to record their counting behavior in a longitudinal study on counting in young children. The notational system allows one to record the various correspondence relations children establish between numerals, their pointing gestures, and objects in their counting activity.

2. Methodology

2.1. Subjects

In the initial sample, 30 2- to 3-year olds were interviewed. Children were re-interviewed on the same set of tasks every two months. Due to various problems in maintaining the initial subject population (e.g., subjects' families changing residence), the original population diminished in size after each testing session. After the third testing session, an additional population of 3-year-olds were included in the sample.

2.2. Materials

The materials included a Sesame Street hand puppet, "The Count" and arrays of miniature plastic animals (between 1 cm and 3 cm tall) affixed to cardboard rectangles. The arrangement of the animal arrays differed on the two general types of tasks that were presented.

2.2.1. Single Array Counting Task

Ten different arrays of animals, two arrays at each of five levels of set size (set sizes 2, 3, 4, 7, and 9) were affixed on cards. Set sizes 2, 3, and 4 were on 8 cm x 20 cm cards, and set sizes 7 and 9 were on 8 cm x 30 cm cards. The types of toy animals varied across cards but not within cards.

2.2.2. Comparison Task

Adjacent comparison. Four cards were used each with two rows of animals. Two of the cards (16 cm x 20 cm) had rows of 3 and 4 animals;

one card had the row of 3 in front of the row of 4, the other card had the rows reversed. The other two cards (16 cm x 30 cm) contained comparable arrays of 7 and 9 animals. The spatial extent of the rows of 3 and 4 were equal as was the spatial extent of rows 7 and 9.

Remote comparison task. Four cards each mounted with a single row of animals was used in the remote task. The array sizes were 3, 4, 7, and 9. Arrays 3 and 4 were mounted on cards 8 cm x 20 cm; arrays 7 and 9 were mounted on cards 8 cm x 30 cm. The arrays of 3 and 4 animals were of equal extent as were the arrays of 7 and 9 animals.

2.2.3. Recording Forms

The child's responses for each of the array cards was recorded on a separate scoring sheet (See appendix). Two forms were used, one for the single array counting task and one for the comparison task. As one individual interviewed the child, a second noted the child's counting on the record sheet.

2.3 Administration Procedures

Children were interviewed individually in the classroom in the presence of two examiners. One examiner introduced the child to a Sesame Street puppet, "the Count," and told the child that the Count worked in a zoo. The Count then told the following story:

"Last night all of my animals ran away. This morning I went all over the city looking for them and put them back in their cages. Now I want to be sure if I have all my animals back. So will you help me count them to see if they are all here?"

2.3.1. Single Array Counting

After the child agreed to help the Count, the first array was presented and the child was asked, "How many (name of animals) are there?" The question was repeated for each array presentation. If the child did not respond or gave only a single number answer, he/she was asked to count the animals. On one presentation of each set size the array was concealed after the child had counted and the child was asked again, "How many (animal name) are there?" The arrays were presented in a random order. Because the subjects were young and inexperienced counters, however, the first array was usually selected from among the smaller set sizes (i.e., 2, 3, or 4 elements).

The child's responses were recorded by the second examiner on the prepared forms. Examiner roles were alternated for each child.

2.3.2. Comparison Task

After the ten single arrays were presented, the child was given a comparison task. Children were presented with two types of comparison tasks in which they were asked to evaluate the relative number of animals in two arrays. In the remote comparison task two linear arrays were separated by a distance that prohibited a direct comparison by sight. In the adjacent comparison task the two linear arrays were aligned on a single card, one beneath the other. The presentation of remote and adjacent tasks were counterbalanced for each child. All children were given remote and adjacent comparison tasks with small set sizes (3 and 4 objects). Only those children who made judgments based on some counting strategy, however, were given the larger set size (7 and 9 objects) tasks.

Adjacent Comparison Task (small set size). The stimulus card was placed on the table in front of the child. Each child received only one display selected at random from the two possible choices.

Adjacent Comparison Task (large set size). The same procedure was followed as with the small set sizes, but using a single stimulus card containing 7 and 9 animals.

Remote Comparison Task (small set size). For each child a linear array of either 3 or 4 toy animals was placed on the table in front of him/her. Another array of 3 or 4 animals was placed on the floor. The relative positions of the two arrays was randomly determined for each child.

Remote Comparison Task (large set size). The same procedure was followed as with the small set sizes, but using arrays of 7 and 9 animals.

Comparison Task Story. After the child was shown an array from either the small set size remote or adjacent comparison task, the Count told the following story:

"It's time to feed the animals now. I want to know how much food I should give them. Over here I have (animal name) and (animal name). Can you tell me if there are more (animal name) or more (animal name) or the same number of both?"

If the child gave no answer to the question or gave an answer without counting, the Count prompted a count by asking, "Can you count and show me?" Following the child's counting response, regardless of

accuracy, the Count repeated, "So, can you tell me if there are more (animal name) or more (animal name) or the same number or both?"

The order in which the names of the animals were said followed two patterns. For the remote comparison task, the name of the animal on the table was always said first. For the adjacent task, the animal appearing closer to the child was said first.

3. Notational System

This section presents a notational system for recording children's counting behavior. Our aim was to record the correspondences children establish between numerals, gestures and objects. To this end, notations were devised to record correspondences between gestures and objects as well as numerals and gestures.

3.1 Gesture to object correspondences

Four types of gestures were recorded, the notations for which are presented below. In addition, one directional descriptor is also presented.

3.1.1 Discrete Pointing Gestures

A pointing gesture consisted of a discrete point to an object. These were represented by a vertical slash (/), underneath the number line¹, which corresponded to the placement of the child's gesture in relation to the actual object. For example, a count in which the child used a discrete gesture for each and every object would appear as:

1	2	3	4	5	6	7
/	/	/	/	/	/	/

3.1.2 Sweeping Gestures

A sweeping motion was recorded as an arrow (—→) extending from

¹ The first count of an array was recorded directly below the number line. If the child counted a second time (either spontaneously or on request) it was recorded directly above the number line.

the beginning to the end point of the gesture. Example:

1 2 3 4 5 6 7

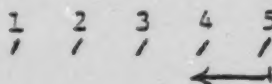

3.1.3. Clumping Gestures

A bracket () was recorded for a clumping gesture in which the child seemed to grasp several objects at a time. Example:

1 2 3 4 5 6 7


3.1.4. Direction of Gesture

The direction of gesturing (if it was other than in a single straight line) was recorded as a vector () and reflected reversals of direction throughout the count. Example: 1 2 3 4 5



3.1.5. Simultaneous Gestures

Gestures made simultaneously to two objects were recorded with two vertical slashes connected by a bracket (). For example, a child who pointed to the first and second objects at the same time would be recorded as: 1 2 3 4 5



3.2. Recording of Number Names

The number names that the child used were recorded in the order in which they were said, using written numerals.

3.2.1. Simultaneous Number Name and Gesture

If the gesture and number name were simultaneous, then the numeral was recorded directly under the gesture mark.

1 2 3 4 5
 / / / / /
 1 2 3 4 5

3.2.2. Non-simultaneous Number Name 2nd Gesture

If a single number name was not accompanied by a single gesture, then the numeral was recorded in its appropriate temporal location in relation to the other number names.

1 2 3 4 5 6 7
 1 2 3 4 5 6 7

In this example the placement of the numeral "4" indicates that it was said between "3" and "5", but that there was no accompanying gesture.

Another example of recording the relationship of more than one number names to a single gesture would be

1 2 3 4 5 6 7
 1,2 3 4 5 6

In this example, the child said the numbers "1", "2" while grasping the first three objects in a single gesture.

3.1.3 Vocal Emphasis of Number Names

Vocal emphasis of the last number name in a count was indicated by a carat ($\wedge$) placed under the numeral.

1 2 3 4 5
 1 2 3 4 5 $\wedge$

In this example the child vocally emphasized the number "5."

3.1.4 Repetition of Number Name After Counting

If the child repeated the last number name after a count, we wrote

the numeral in quotes (e.g. "5") to the right of the actual count.

1 2 3 4 5
1 2 3 4 5 "5"

3.1.5. Statement of Number Name Before Counting

If the child gave a single number name before the count the number was recorded below the count recording and placed in quotation marks. In the example,

1 2 3 4 5 6
1 2
"2"

the child first said the number "2" and then proceeded to count.

4. Scoring

In order to quantify our observations of responses we devised a scoring system. The scoring categories are grouped into several general areas. These include: The use of number names, the use of gesture, the coordination of names to gestures, the coordinations of gestures to objects, the understanding and use of cardinal value, and the overall performance.

4.1. Number Names

These categories apply to the child's use of number names.

4.1.1. Conventional Order: Does child use conventional order of numerals in his count? (VIII)

0 - Child uses no number names in count (regardless of accuracy in counting) or uses words other than number names. This score is not used for a mix of number names and numerals.

1 - Child uses conventional order (beginning with 1) regardless of counting accuracy.

2 - Child uses unconventional order regardless of accuracy (Refers only to number names).

1	2	3	4	5	6	7
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{10}$

3 - Child uses a list of words that are not number names, e.g., mommy, daddy, mommy, daddy.

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4.1.2. Unconventional order: If child uses an unconventional order
of numerals the list of numerals is recorded. (IX)

4.1.3. The Number of Number Names the Child Uses. (XV)

1 2 3 4 5 6 7
1 2 3 4 5 6 7

Score = 7

1 2 3 4 5 6 7
1 2 3 4 5 6 7

Score = 7

If the child gives only a cardinal number and
does not count, or if there is an aborted count

Score = 99.

4.2. Gesture. These categories apply to the children's use of
gesturing in their counting.

4.2.1. Direction of Sequence of Gesturing. (VII)

0 - Child's gesturing progresses in only one direction
regardless of starting point.

1 - Child's gesturing reverses direction one time.

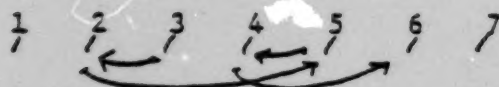
1 2 3 4 5 6 7
1 2 3 4 5 6 7

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2 - Child's gesturing reverses direction twice.



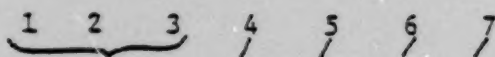
3 - Child's gesturing reverses direction more than once.



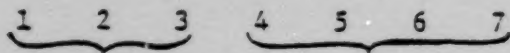
4.2.2 Clumping: Child touches more than one object with a single gesture. (XII)

0 - Child doesn't use clumping.

1 - Child clumps one time.



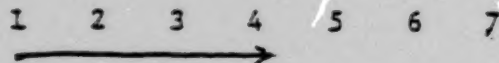
2 - Child clumps more than once.



4.2.3 Sweeping: In a global gesture the child denotes several object by passing over them without touching them. (XIII)

0 - Does not occur.

1 - Child makes one sweeping gesture.



2 - Child makes more than one sweeping gesture.



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4.3. Names to Gestures: These categories apply to the child's coordination of number names with gestures.

4.3.1. Name without Gesture: The child says a number name without an accompanying gesture. (V)

0 - Does not occur.

1 - The child says a number name without an accompanying gesture one time.

1 2 3 4 5 6 7
/ / / / / / /
1 2 3,4 5 6 7 8

2 - The child says a number name without an accompanying gesture more than one time.

1 2 3 4 5 6 7
/ / / / / / /
1 2 3 4 5 6 7 8,9

3 - The child uses no gestures.

1 2 3 4 5 6 7
/ / / / / / /
1 2 3 4 5 6 7

4.3.2 Gestures without Name: The child gestures without an accompanying number name. (VI)

0 - Does not occur.

1 - The child gestures without any accompanying number name, one time.

1 2 3 4 5 6 7
/ / / / / / /
1 2 3 4 5 6 7

- 2 - The child gestures without an accompanying number name more than one time.

1 2 3 4 5 6 7
/ / / / / / /

- 3 - The child gestures without using any number or other names.

1 2 3 4 5 6 7
/ / / / / / /

4.3.3. Discrete Gesture Without Touching Object: Child uses a discrete gesture without touching an object; e.g., between objects, or in the air above the array.

- 0 - Does not occur.

- 1 - One of the child's gestures does not touch an object.

1 2 3 4 5 6 7
/ / / / / / /

- 2 - More than one of the child's gestures does not touch an object.

1 2 3 4 5 6 7
/ / / / / / /

4.4. Gestures to Object.

These categories apply to the child's coordination of gestures with the objects in the array.

4.4.1. Child Gestures More Than Once To The Same Object in a Count. (III)

0 - Does not occur.

1 - The child points twice to one of the objects.

1 2 3 4 5 6 7
 1 2 34 5 6 7 8

2 - The child gestures more than once to a single
 object several times.

1 2 3 4 5 6 7
 1 2 34 5 67 8 9

4.4.2. Skipping Objects: The child does not point to every object in the array; i.e., he skips objects. (IV)

0 - Does not occur.

1 - The child skips only one object in gesturing.

1 2 3 4 5 6 7
 1 2 3 4 5 6

2 - The child skips more than one object in gesturing.

1 2 3 4 5 6 7
 1 2 3 4 5

3 - In the case of two-object arrays, the child points
 simultaneously to both objects. Even though there
 is no skipping of objects, the child fails to pro-
 vide a discrete gesture for each object.

1 2
 1 1

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4.5. Cardinality

These categories apply to the child's understanding of cardinal value.

4.5.1. Spontaneous Cardinal Before Count: Spontaneous use of a single

number name to represent the whole array prior to counting the array. (X)

- 0 - Does not occur.
- 1 - Child offers a single number name that corresponds to the actual number of objects in the array.
- 2 - Child offers a single number name that is inaccurate but within one digit of the correct number of objects in the array.
- 3 - Child offers a single number name that is inaccurate by more than one.
- 4 - After counting, the child emphasizes the final number name in his count vocally.

4.5.2. Spontaneous Cardinal After Count: Spontaneous use of a single

number name to represent the whole array after counting. (XI)

- 0 - Does not occur.
- 1 - Child offers a single number name that corresponds to the actual number of objects in the array.

- 2 - Child offers a single number name that is inaccurate but is within one digit of the correct number.
- 3 - Child offers a single number name that is inaccurate by more than one.
- 4 - Child offers a single number name that is the final number name in the child's count.

4.5.3. Elicited Cardinal

Of the two arrays presented for each of the set sizes, one is followed by the elicited cardinal task. After counting the array, the child is asked, "How many are there?" and responses are recorded.

- 0 - Child is not asked for a cardinal response on a presentation.
- 1 - In response to the question, child offers a single number name corresponding to the actual number of objects in the array. Example:

1	2	3	4	5	6	7
$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$

Q: Child: "7"

- 2 - Child offers a single number corresponding to the final number in his count, although the count was inaccurate or used an unconventional number list.

Example:

1	2	3	4	5	6	7
$\frac{1}{1}$	$\frac{2}{2}$		$\frac{3}{3}$	$\frac{4}{4}$		$\frac{5}{5}$

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Q: Child: "5"

- 3 - Child offers a chain of number with the last number name corresponding to the final number of his previous count, even if that count had errors. Example:

1	2	3	4	5	6	7
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>

Q: Child: "5, 6, 7"

- 4 - Child offers a chain of number names where the last number name does not correspond to the final number of his count. Example:

1	2	3	4	5	6	7
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>

Q: Child: "8, 9, 10"

- 5 - Child offers a single number name that does not correspond to the number of objects in the array or to his own count. Example:

1	2	3	4	5	6	7
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>8</u>

Q: Child: "9"

- 6 - Child makes no response or an inappropriate response (answer contains no number names). Example:

1	2	3	4	5	6	7
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>

Q: Child: "All rabbits"

4.5 Overall Accuracy

These categories describe overall performance.

4.5.1. Overall Accuracy. (I)

0 - Perfect count: Child accurately coordinates conventional number names and gestures to each and every object, once and only once. Example:

1	2	3	4	5	6	7
/	/	/	/	/	/	/
1	2	3	4	5	6	7

1 - Not perfect: Child makes any one or combination of errors in coordinating number name, gesture, and objects. Example:

1	2	3	4	5	6	7
/		/	//	/	/	/
1		2	34	5	6	

2 - One-to-one unconventional: Using an unconventional number list, child accurately coordinates number names and gestures to each and every object in an array, once and only once. Example:

1	2	3	4	5	6	7
/	/	/	/	/	/	/
3	4	7	8	9	10	14

3 - Inappropriate: Child neither counts nor uses a single number name. Example:

1	2	3	4	5	6	7
/	/		/		/	
tail	- tail		tail		tail	

- 4 - Cardinal only: Child offers a single number name without pointing to any objects. The number may or may not be accurate. Example:

1 2 3 4 5 6 7

Child: "2"

- 5 - Accurate count without touching every object:

Using a conventional number list the child arrives at the correct final number, although he does not touch each and every object. This category represents the performance of children who can visually keep track of their place in an array without having to point to all objects. Example:

1 2 3 4 5 6 7
1 2 3 4 5 6 7

4.6.2. Number of Repeats: This category represents the number of times the child spontaneously repeats a count on the same array. Repeated counts at the request of the experimenter are not included here. (II)

0 - No repeats: Child counts only once.

1 - Repeats: Child makes 2 or more complete counts on the same array. This score is given only on the first count of any array presentation.

2 - Aborted count: Child makes 2 or more counts, one or more of which is not completed. This score is

given on all incomplete repeated counts. Example:

¹1 ²2 ³3 ⁴4 ⁵5 ⁶6 ⁷7
1 2 3 4 5 6 7
1 2

PROBLEM CODES

In this section we will present examples of how we would go about scoring some recorded counts which might seem problematical. Each example will be followed by an explanation of the child's behavior, the scoring of the relevant categories, and our rationale for scoring it in that way.

Several problems arose in attempting to score performances in which the child "clumped" together several objects with one gesture.

Example 1

1 2 3 4 5 6 7 8 9
 └───┘ └───┘
 1 2,3

In this example the child grabbed the first three objects while saying "1" and then grabbed the last three objects while saying "2", "3"

We scored this as follows:

4.4.1. (III) Child gestures more than once to same object.

0 - Does not occur.

4.4.2. (IV) Child does not point to every object in array.

2 - Skips more than one object.

4.3.1. (V) More names than a single gesture.

1 - The child says a number name without gesture, one time.

When a child uses clumping gesture we decided that in the case where 3 or more objects were clumped it would be scored as "skipping" an object 4.4.2. That is, a clump is considered as touching the first and last object under the clump — the objects between the first and last object in the clump are "skipped." Also each clumping gesture would be considered as a single gesture which accounts for scoring of category 4.3.1., the child said "2, 3" while using only one gesture.

We found that "clumping" two objects presented a problem. We considered that scoring a clump as a "skip" was harder to justify when the child grouped by two's. Each object could be easily touched with one gesture and the child's intent may not have been to skip an object. Therefore if the child "clumped" two objects it was not scored as a skip (i.e., 4.4.2., is scored as 0).

Example 2

1	2	3	4	5	6	7
1, 2, 3			4	5	6	7

We scored this as follows:

4.4.2. (IV) Child does not point to every object in array.

1 - Skips one object.

4.3.1. (V) More names than gestures.

2 - Says more number names than gestures.

Example 3

1 2 3 4 5 6 7
 ──────────→

1, 2, 3, 4

Sweeping gestures were all scored as skipping objects - that is each object was not referred to with a discrete gesture. Each sweeping motion is considered a single gesture.

We scored this as follows:

4.4.2 (IV) Child does not point to every object in array.

2 - Skips more than one object.

4.3.1. (V) More names than a gesture.

2 - Says more names than gestures.

Example 4

1 2 3 4 5 6 7
 → 4 5 6 7
 1, 2, 3

If the number of numerals corresponds to the number of objects that are subsumed by a clumping or sweeping gesture, and the rest of the count is accurate, then the overall accuracy of this response is scored as a "5" in category 4.6.1 i.e., an accurate count while not pointing to each object.

Example 5

1 2 3 4 5 6 7
/ / / / / // /
/ 2 3, 4 5 6, 7 8

Scoring:

4.3.1.(V) Number name without accompanying gesture.

1 - Occurs once.

4.3.2.(V) Gesture without accompanying name.

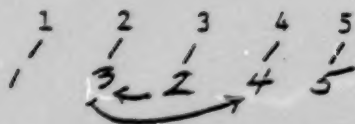
1 - Occurs once.

4.4.1.(III) Gestures more than once to same object.

1 - Occurs once.

4.4.2.(IV) Does not point to every object :

1 - Occurs once.

Example 6

In this example the child does not gesture to the objects in sequential order, but changes direction. He does, however, gesture to every object.

Scoring

4.1.1. (VIII) Use of conventional order of numerals.

1 - Uses conventional order.

4.2.1. (VII) Direction of sequence of count.

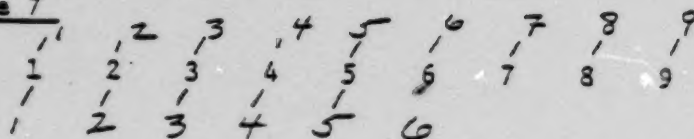
2 - Gesturing reverses direction twice.

4.4.2. (IV) Child does not point to every object.

0 - Does not occur.

Category 4.4.2., is the most problematical one. As you see, though the child did skip over an object in the course of his count he did come back to it later in the count so that all object were gestered to.

Example 7



When a child's initial count is interrupted but is followed by a complete count (whether accurate or inaccurate) the original count is scored as follows:

Accuracy is recorded only for that portion of the array actually counted. In addition the original count is scored as an aborted count. Any other errors or responses would also be included in the score.

In the above array of 9 objects, for example, the child counted up to object 5, the last object to which he pointed. His score, therefore, would be:

- 4.6.1.(I) 1 - Inaccurate count [because of the extra number name]
- 4.6.2.(II) 4 - Aborted count.
- 4.3.1.(V) 1 - 0 The child says a number name without an accompanying gesture.
- 4.1.1.(VIII) 1 - Conventional count.
- 4.1.3.(XV) 99 - Indeterminant number of numerals.

Some additional problems

Determining whether a child was coordinating gesture with number name was sometimes difficult. We at times would see a delay between the gesture and the name which seemed to be a lack of physical coordination rather than a lack of understanding that names and gestures should be

coordinated. Because of this we let our intuition of what we thought the child's actual intention was guide us in recording name/gesture coordination.

A second problem concerned what we called "holding gestures." We would see children hesitate during a count and then resume their count by gesturing again to the object they had last counted and repeating its number name. If we considered this a resumption of the count and the double point a "holding gesture" in the sense of holding the child's place, we would not score it as more than one gesture for an object, but rather as a perfect count.

Finally, if a child offers a spontaneous cardinal value for the array before he or she counts, and then proceeds to count the array, we scored this as a single count.

Section V.

Papua New Guinea Research (on numerical concepts other
than counting/conservation relations)

When Fourth Can Precede Second:

A Developmental Analysis of an Indigenous Numeration System

Among Ponam Islanders in Papua New Guinea

Geoffrey B. Saxe

The Graduate Center of the

City University of New York

Running head: When Fourth Can Precede Second

BIOGRAPHICAL SKETCH

Geoffrey B. Saxe received his Ph.D. from the University of California at Berkeley and is currently an Assistant Professor in the Department of Educational Psychology at the Graduate Center of the City University of New York, 33 West 42nd Street, New York, New York. Using a comparative approach to study cognition, he is engaged in research on cognitive development among Western, non-Western, and atypical ("learning disabled" and brain damaged) populations.

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Department of Educational Psychology, The Graduate Center of the City University of New York, 33 West 42nd Street, New York, New York 10036.

When Fourth Can Precede Second

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Abstract

This paper presents a developmental analysis of children's understanding of an indigenous birth order naming system used in a remote Papua New Guinea island community (Ponam Island). The naming system is organized such that boys are given one set of names depending upon their birth order relation to other boys of a family, and girls are given a separate set of names depending upon their birth order relation to other girls of a family. The names are invariant across families. On the basis of Piaget's theory, it was hypothesized that developmental changes in operational reasoning would be necessary for children to understand the determinate and indeterminate age relations implied by birth order names within and across sex. To test this formulation, 29 Ponam islanders were interviewed who ranged in age from 8 to 23 years. The findings supported the general formulation.

When Fourth Can Precede Second

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When Fourth Can Precede Second:

A Developmental Analysis of an Indigenous Numeration System
Among Ponam Islanders in Papua New Guinea

Notational systems have been invented by cultural groups to represent logical and numerical relations. This paper describes a notational system for birth order relations used by residents of Ponam, a remote island community in Papua New Guinea, and presents an analysis of developmental changes in island children's understanding of the structure of this notational system. The analysis is guided by Piaget's general formulation of universal developmental changes in the understanding of ordinal relations (Piaget, 1970, 1972).

PONAM ISLAND AND ITS BIRTH ORDER NAMING SYSTEM

Ponam island is located in the Admiralty chain in Manus Providence of Papua New Guinea. The island is about 3/4 miles long and about 1/4 mile in breadth at its widest point. Fish and tropical fruit are the major sources of subsistence, and there is weekly trade with other island populations for unavailable goods (e.g., sago, the major staple). There is a Christian church on the island, and a government elementary school.

The birth order naming system used on Ponam is organized such that boys are given one of a set of eight names which signifies their birth order relation to other boys of their family, and girls are given one of a different set of eight names which signifies their birth order relation to other girls of their family. These names are invariant across families and within each sex.

This notational system for birth order has a number of interesting logical properties. First, within a given sex, the relative ages of two children of a family can be determined if their birth order names are known. For example, Tol, the first born boy, must be older than Sebat, the fourth born boy. Similarly, Pindaluh, the first born girl, must be older than Pindrnew, the fourth born girl. Another interesting property of this system is that, although the age relations between same-sexed children of a family are determinate, the age relations between opposite sexed children of a family are indeterminate. For example, one could never know the age relation between Tol, the first born boy, and Pitatah, the second born girl, simply by reference to their names. Although Tol is a first born boy, he may or may not be older than Pitatah, the second born girl, since two or more girls of a family could be born before the first boy. If this occurred, Pitatah would be older than Tol.

UNIVERSAL CHANGES IN THE CHILD'S UNDERSTANDING OF ORDINAL RELATIONS

According to Piaget's theory, the ability to coordinate ordinal relations undergoes a universal set of qualitative changes over the course of development. These changes are considered the product of a dialectical process in which increasingly more powerful means of ordering relations are elaborated as a result of interactions with physical and social reality. On the basis of research with Western children, Piaget argued that young children are unable to coordinate two relations of difference (Piaget, 1952). As a consequence, children cannot order a series of ten sticks from shortest to tallest, nor understand certain forms of transitive inference (e.g.,

if $A < B$, and $B < C$, then $A < C$), nor understand certain forms of reciprocal relations (e.g., if A is taller than B, then B is shorter than A). During middle childhood, children achieve the ability to coordinate two relations of difference, and use this ability to construct these various forms of ordinal concepts. Among two series of elements, however, children are still unable to generate systematically all possible combinations and permutations of order relations for sets of elements. For instance, given the two series A,B,C,D, and a,b,c,d, children are unable to use a combinatorial system such that, in principle, all possible orders can be considered without violating either series, (e.g., ABCDabcd, ...ABabccDd, ...abcdABCD). It is not until adolescence that this ability is achieved and, Piaget argued, that it is this form of coordination of possibilities that provides a cognitive basis for hypothetico-deductive reasoning (Inhelder & Piaget, 1956).

DEVELOPMENTAL CHANGES IN THE PONAM CHILD'S
UNDERSTANDING OF THE BIRTH ORDER NAMING SYSTEM

Piaget's analysis suggests that Ponam children's understanding of the logical properties of their birth order naming system should undergo particular forms of qualitative changes over increasing age. Specifically, although young children may know the birth order names, they should not understand the implications of these names for relative age relations of individuals of a family (relative birth order being the reciprocal of relative age). Somewhat older children, who have achieved the ability to coordinate two relations of difference, should understand this inverse correspondence between birth order and age within sex. However, in order to understand the indeterminacy of age relations across sex based on birth order names, children would need to be able to generate possible or hypothetical combinations of birth orders of a family. For example, in order to judge correctly whether Tol, first born boy, is older than Pitatah, second born girl, one must generate and consider at least two sets of birth orders from all possible birth orders, one set in which Tol precedes Pitatah, and one in which Tol follows Pitatah. As a consequence, although children may understand the determinacy of relative age relations within sex based on birth order names, many of these children should not understand the indeterminacy of age relations across sex. Finally, at maturity, in addition to the ability to understand the determinacy of age relations within sex, children should be able to

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generate and consider different possible classes of birth orders from all of the possible birth orders. This would provide the cognitive basis for the judgment that across sex age relations are indeterminate on the basis of the birth order names.

In order to test this formulation with Ponam children, a methodology was developed that permitted an analysis of children's reasoning about the age relations between individuals that were implied by birth order names. In addition, Piaget's seriation of sticks task was also adapted and administered to children in order to link early developmental changes in reasoning about logical properties of the birth order system with an established developmental sequence in children's ability to coordinate ordinal relations.

MethodPreliminary Procedure

The method employed in the present study was the product of a series of preliminary phases. First, a period of time was spent becoming familiar with the structure of the birth order naming system with several informants. Second, on the basis of these interviews, task procedures were designed that would elicit children's understanding of logical properties of the system. These procedures were then translated from English to Ponam with the aid of two informants who were fluent in English and Ponam. A back translation procedure was used to accomplish this translation. In this procedure, a sentence, or a series of sentences, was stated in English and the first informant translated the sentences into

Ponam. Only the first informant's Ponam translation was tape recorded. After the translation was completed, the informant was required to back translate the tape recording into English. When necessary, parts of the initial translation were altered until the procedure yielded an accurate back translation from Ponam. As a further check, the second informant was required to back translate the first informants final translation into English. Third, Ponam children at different age levels were interviewed to determine in what ways the procedures could be improved. Fourth, the back translations process was repeated using the modified procedures. These pilot phases led to the procedure used in the study described below.

Subjects

Children at three age levels were interviewed: ten 8- to 11-year olds (2 years of schooling, mean age = 9.9 years, standard deviation = .88 years); ten 12- to 15-year olds (5 years of schooling, mean age = 13.1 years, standard deviation = .99 years); and nine 16- to 23-year-olds (at least 6 years of schooling, mean age = 19.2 years, standard deviation = 2.7 years). Subjects' ages were obtained from missionary birth records. Subjects were recruited to participate from the school on the island and by asking individuals, who were not enrolled in school, through the informants.

Birth order task. The materials used in the birth order task consisted of nine cards containing pictures of stick figures which were drawn by a native informant (see figure 1). One card was about 10 cm x 22 cm and contained stick figures of a male and a female. The other eight cards were each about 4 cm x 5 cm, four of which contained identical stick figures of

males and four of which contained identical stick figures of females. On each of the two sets of four male and female stick figure cards, a sex appropriate birth order name was printed ranging from first to fourth born, with the appropriate numeral designation, "1", "2", "3", or "4".¹

Insert Figure 1 about here

Seriation of sticks. Twelve sticks of about 1 cm diameter bamboo were cut into pieces ranging from about 10 cm to about 22 cm. Sticks differed in length by about 1 cm. A large notebook was used as a screening device.

Procedures

The administration of the birth order task always preceded the administration of the seriation of sticks task.

Birth order task. This task was administered in three phases: a warm-up/subject selection phase, a family generation phase, and an interview question phase.

(a) Subject selection/Warm-up phase. Children were asked if they knew the birth order names and then asked to tell the first four for each sex. If children did not know the names, they were excluded from the study. Children were then presented with the large card containing the male and female and told that the stick figures on the card represent a man and a wife. Children were then asked, "What is the name of the first (second, third, or fourth) born boy (or girl)?" As children responded, the appropriate one of the eight smaller cards was placed on the table as shown in Figure 1. Once this procedure was completed, the

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next phase began.

(b) Family generation phase. This phase had two purposes. The first was to call children's attention to the fact that boys and girls of a family could be born in different orders. The second was to determine the facility with which children generated these different possible orders.

During this phase, the cards were presented as shown in Figure 2. Children were told to make up a family in a single line in front of the husband and wife so that all of the children had their proper birth order names. Children were instructed to put the first born directly in front of the husband and wife, the second born next, until they finished placing and naming all of the cards. Once children had completed a family, the cards were returned to their initial position (Figure 1), and children were asked to create a new family, having a new series of birth order names. The same procedure was repeated a third time. If on the second or third occasion children repeated a previous family order, children were told that they had to make a new family, in a new order. If children could not produce a second or third birth order, the interviewer helped them. Once three orders were completed, the cards were returned to their initial configuration (Figure 1), and children were interviewed about age relations between members of a family implied by the birth order names.

Insert Figure 2 about here

(c) Interview question phase. With the cards in their initial configuration (Figure 1), children were asked a series of four questions of the following form: "Who is older, _____ or _____? How do you know? Could _____ ever be older than _____? How do you know? If children responded affirmatively, they were required to construct a family with the cards arranged demonstrating this possibility. The first and third repetitions of the question were within sex comparisons, and the second and fourth repetitions were between sex comparisons. Children were assigned to one of three levels for each question type which depended upon their number of correct responses. For each question type, children were assigned to Level I if they did not achieve any correct answers, to Level II if they achieved one correct answer, and to Level III if they achieved two correct answers.

Seriation of sticks. Children were presented with the set of twelve sticks and asked to order them from the shortest to the tallest. Children's productions were recorded. If children produced a perfect series, whether by trial and error or by a systematic method, the sticks were rescrambled. Children were then told to hand the interviewer one stick at a time, proceeding from the shortest to the longest. After each successive stick, they were told that the interviewer was going to put each stick, one after another behind a screen and wanted to make the sticks line up perfectly. Children were told that they had to be sure they were handing the correct stick, or else they would not achieve the correct order.

Children's performances were categorized as one of three levels that were adaptations of Piaget's scoring criteria. Level I was defined as a

failure to produce a regular series (at least two or more sticks in an incorrect order.) Level II was defined as an adequate seriation, but at least one error in the second phase of the task. Level III was defined as a successful performance across both phases of the task.

Results

In general, the findings were consistent with the hypotheses. Table 1 contains a frequency distribution of the levels of children's responses for each age group on the within- and between-sex questions. With age, there was an increase in likelihood of understanding the determinate relations for the within sex questions, although this trend did not achieve statistical significance (collapsing response Levels I and II, Fisher exact tests for the 10-year-old/13-year-old age group comparison indicated $p = .35$, and for the 13-year-old/19-year-old age group comparison indicated $p = .26$). With age, there was an increase in likelihood of understanding the indeterminate relation for the between sex questions, and most children did not achieve this understanding until late adolescence (collapsing Levels I and II, Fisher exact tests for the 10-year-old/13-year-old group indicated $p = .50$, and for the 13-year-old/19-year-old group comparison indicated $p = .005$).

Insert Table 1 about here

The differences in the age norms for understanding the determinate and indeterminate properties of the within- and between-sex questions suggests an invariant sequence in children's understanding of these relations. Stronger

evidence for such an invariance claim, however, is revealed by an analysis of children's profiles across the within- and between-sex questions. Indeed, of the entire sample, none of the children achieved higher levels of performance on the between-sex questions than on the within-sex questions (sign test, $N = 19$, $x = 0$, $p < .001$).

Table 2 contains the distribution of children's scores on the seriation of sticks task. As expected, there is a developmental trend in children's ability to produce an adequate series (collapsing Level I and Level II scores, Fisher exact test for the 10-year-old/13-year-old comparison indicated $p = .33$, and for the 13-year-old/19-year-old comparison indicated $p = .05$). The distribution of children's performances on this task tended to parallel their understanding of the within-sex questions. In general, in the youngest two age groups, children demonstrated some ability to coordinate a series of ordinal relations, and in the oldest age group, all children had developed this ability. Moreover, 3 of the 5 children who did not respond at the highest level on the within-sex questions also did not respond at the highest level on the seriation of sticks task.

Insert Table 2 about here

Discussion

The results support the general formulation presented in the introduction. Children's understanding of the logical properties of their indigenous birth order naming system is not a product of simply learning the conventional list of names in the prescribed sequence. Rather, the evidence supports the formulation that qualitative changes occur in children's ability to coordinate ordinal relations, and that changes in this ability underlie children's understanding of the logical properties of the system. Thus, in the youngest age group, some children did not understand the reciprocal relation between relative birth order and relative age, but in the two oldest age groups this relation was mastered. A clear developmental trend was also found in the child's understanding of the indeterminacy of age relations in across sex birth order name comparisons. Despite the fact that all children generated various orders of birth in the "family generation phase," they did not appreciate the significance of these various possible orders when considering age relations implied by the birth order names across sexes. It was not until late adolescence that a majority of the children understood that one could not determine relative ages of children across sex given the birth order names of a male and a female.

Subject's responses can be understood as conforming to Piaget's general stages of cognitive development. The determinate questions required the coordination of two series of asymmetrical relations—a criterion of Piaget's stage of concrete operational thought. Subjects demonstrated this ability by inferring the appropriate age relation of two same-sexed children on the

basis of their birth order names. The indeterminate questions, in addition to the coordination of two series of asymmetrical relations, required the use of a elementary form of combinatorial reasoning—a criterion of formal thought. Although this task did not require the subjects to systematically generate all possible combinations—the advanced substage of formal operations, the fact that subjects had to generate and make inferences on the basis of hypothetical combinations of birth order suggests the emergence of a combinatorial system and is consistent with Piaget's description of the early manifestation of formal thought (Inhelder & Piaget, 1956; Piaget, 1972). As evidence that this form of reasoning was the source of subjects' judgments of indeterminacy, all subjects who were classified at the advanced response level had to generate appropriate permutations of the birth order cards to "prove" that the across sex age relation was indeterminate.

Cross cultural approaches to the study of cognition take one of two forms (Berry & Dasen, 1974; Pike, 1966; Sturtevant, 1964). "Etic" approaches use categories of analysis which are applied across cultural settings, whereas "emic" approaches use categories of analysis which are constructed within a particular cultural setting and are applicable only to that setting. There are advantages and disadvantages to each approach. Emic approaches insure comparability of findings across cultural groups; however, they forfeit an analysis of the unique character of any particular group by fitting groups into pre-established categories. Emic approaches capture the unique way in which a particular group organizes reality; however, comparisons across cultural groups are, at best, difficult, if not logically impossible. The

present study demonstrates, on a small scale, how etic and emic approaches can be coordinated in a research program such that some of the advantages of both are gained. By focusing on an indigenous knowledge system, the approach is faithful to a unique construction of Ponam culture, and, at the same time, by using a theoretical framework which makes substantive claims about universal processes in the development of intelligence, the analysis has produced findings which are comparable to findings on cognitive development in other cultural groups.

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Footnotes

¹Children did not have difficulty identifying the cards with specific birth order names. Although not all of the children could read the birth order names, all children could read the numerals on the cards and identify which cards contained boys and which cards contained girls.

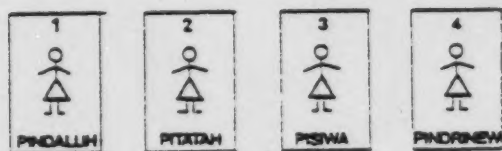
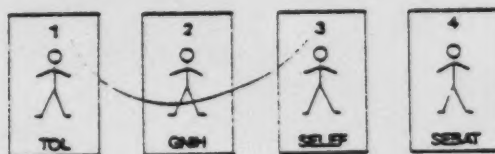
Table 1

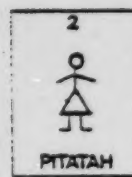
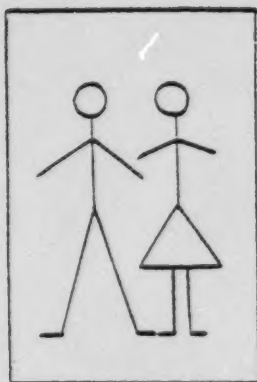
Frequency Distribution of Children's Response Levels
on Within- and Between-sex Questions for Three Age Groups

Age Group	Question Type					
	Within-sex			Between-sex		
	Level I	Level II	Level III	Level I	Level II	Level III
10 years	1	2	7	10	0	0
13 years	0	2	8	8	1	1
19 years	0	0	9	1	1	7

Table 2
Frequency Distribution of Children's Levels
of Performance on Seriation Task for Three Age Groups

Age Group	Seriation Performance		
	Level I	Level II	Level III
10 years	3	3	4
13 years	0	4	6
19 years	0	0	9





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THE CHANGING FORM OF NUMERICAL THOUGHT AS A FUNCTION OF CONTACT
WITH CURRENCY AMONG THE OKSAPMIN OF PAPUA NEW GUINEA

Geoffrey B. Saxe

555-78-1273

The Graduate Center of the City University of New York

33 West 42nd Street, New York, N.Y. 10036

Slides

THE CHANGING FORM OF NUMERICAL THOUGHT AS A FUNCTION
OF CONTACT WITH CURRENCY AMONG THE OKSAPMIN

Geoffrey B. Saxe CUNY/Graduate Center

The purpose of this paper is to analyze new forms of numerical thought which are emerging among the Oksapmin, a remote group in Papua New Guinea, as a function of their contact with Western currency.

The indigenous numeration system used by the Oksapmin consists of 27 body parts, beginning from the thumb of one hand, and progressing around the upper periphery of the body to the little finger of the opposite hand. Until the recent introduction of currency to the area, there was little, if any, social context to engage in arithmetic computations in the community. In the recent past, a tea plantation has begun to fly individuals out of the area to work (100 miles away). Upon their return, some people construct tiny trade stores and do business in the community.

In order to determine the way in which the degree of involvement with economic exchange is influencing people's understanding of number, four groups of people were interviewed about their solution strategies to arithmetic problems. The groups differed in their levels of engagement with currency (trade store owners, plantation goers, young adults, older adults). The findings indicated that individual's are creating novel computational strategies with the indigenous system depending upon their degree of involvement with economic exchange. The computational strategies reflect qualitative shifts in the way in which the Oksapmin use and understand numerical relations.

SUBMITTER'S NAME: Geoffrey B. Saxe

SUBMITTER'S SOCIAL SECURITY NUMBER: 555-78-1273

The Changing Form of Numerical Thought As A Function of Contact

With Currency Among the Oksapmin of Papua New Guinea

555-78-1273

The purpose of this paper is to analyze new forms of arithmetic computation that are emerging among the Oksapmin, a recently contacted cultural group in Papua New Guinea. The major thesis is that "primitive" forms of numerical thought change to more abstract forms as a result of people's cognitive adaptations to new types of problems for which the primitive forms are ill-suited. The new adaptations in numerical thought are not a simple replacement of old forms with new ones; rather, the change in numerical thought is reflected in progressively more efficient means of employing the traditional forms to solve new problems. In the present paper, the concern is to document a progression of new ways of arithmetic thinking among the Oksapmin which are emerging as a function of people's degree of involvement with a new type of economic exchange, that involving Western currency.

The indigenous numeration system used by the Oksapmin consists of twenty-seven body parts beginning with the thumb on one hand and progressing around the upper periphery of the body to the little finger on the opposite hand (see Figure). Traditionally, the Oksapmin rarely needed to perform arithmetic calculations; however, in recent years, a tea plantation located 100 miles from the region has flown men out of the region to work, and men return with currency. Some open tiny trade stores and do business by selling cans of fish and bags of rice.

In order to determine how the degree of engagement with economic exchange with currency affects the way in which people use the indigenous system to

produce arithmetic computations, four groups of Oksapmin adults were interviewed who differed in their degree of engagement with economic transactions. These groups consisted of trade store owners (n = 20), men who had worked at the plantation but did not own trade store (n = 20), 25-35 year old men and women (no plantation or trade store experience), 45-60 year old men and women (no plantation or trade store experience).

Individuals were each administered eight arithmetic problems individually involving one shilling coins. In four of the problems (two addition and two subtraction) shillings were present, and in four problems (two addition and two subtraction) shillings were not present and only appropriate body parts were indicated. All problems were presented using a standard procedure, and, when needed, a set of standard probe questions were used to elicit covert strategies.

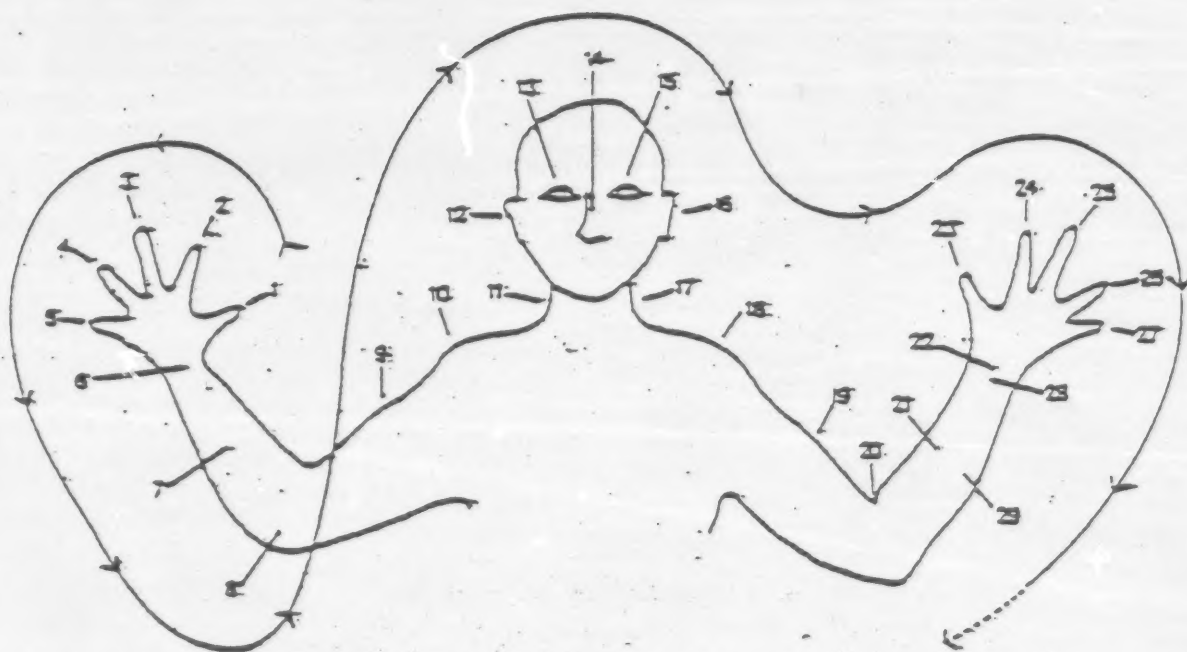
The results are currently being analyzed, however, preliminary analyses indicate dramatic differences in body part strategies which people use across groups. The groups which have greater degrees of contact with economic exchange use more efficient and differentiated means of solving the problems than the groups that have lesser degrees of contact. The following are exemplars of three prototypical strategies which individuals used across three of the four groups in response to the problem, "you have 16 shillings (ear on other side of the body—both stated and pointed to) and spend 7 shillings at the trade store (forearm—both stated and pointed to). How many do you have left?" Traditional adult: Enumerates thumb (1) to forearm (7), and says he spends these. Then, says he has ^{inner elbow (5th)} shoulder (10th), neck (11th), ear (12th)...ear on other side (16th) left. He will not give a precise numeric representation, despite probing. Plantation goer: Calls

ear on other side of body the thumb (16 = 1), calls eye the first finger (15 = 2), calls nose the second finger (14 = 3), ... calls shoulder the forearm (10 = 7) and says that there are bicep (9) left. Tradestore owner: Says that 16 shillings can be called shoulder on left arm (10) and wrist on the right arm (6), i.e., $10+6=16$. From the thumb (1) of the left arm up to the forearm (7) are spent, leaving inner elbow (8th), bicep (9th) and shoulder (10th) on the left. Add these body parts (inner elbow (8th), bicep (9th), and shoulder (10th) on the left arm) to the wrist (6) on the right arm, and the result is the bicep (9).

In the paper, it will be argued that the three forms of strategies cited above constitute a progression from a type of strategy in which individuals do not differentiate between body parts and numerical representations (traditional people) to a type in which numerical representations are distinguished from body parts in that one body part can be called by the name of another body part in an arithmetic operation (plantation goers and tradestore owners). Moreover, it is of interest that the prototypical strategy of the tradestore owner but not the plantation goer reflects the base structure of the system of currency. Thus, larger numbers are broken down into units of tens and ones (or twenties and ones), a base principle reflected in the structure of Papua New Guinea currency, but not in the traditional Oksapmin numeration system.

In sum, the present paper documents the way in which a "primitive" cultural group is adapting to a new cultural press—economic exchange with Western currency—and, in so doing, inventing new ways of conceiving of numerical relations.

Figure 1



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SOCIAL CHANGE AND COGNITIVE GROWTH: THE INVENTION OF BODY
PART ARITHMETIC ALGORITHMS AMONG OKSAPMIN CHILDREN OF PAPUA NEW GUINEA

Geoffrey B. Saxe

555-78-1273

The Graduate Center of the City University of New York

33 West 42nd Street, New York, N.Y. 10036

Slides

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SOCIAL CHANGE AND COGNITIVE GROWTH: THE INVENTION OF
BODY PART ARITHMETIC ALGORITHMS BY OKSAPMIN CHILDREN
Geoffrey B. Saxe CUNY/Graduate Center.

The purpose of this paper is to analyze new arithmetic algorithms that are emerging among Oksapmin children in a remote area of Papua New Guinea.

The indigenous numeration system of the Oksapmin consists of naming and/or pointing to 27 body parts beginning from the thumb on one hand, following the upper periphery of the body, and ending at the little finger of the opposite hand. There is little, if any, social context in the traditional community to engage in arithmetic computations; however, a bush elementary school was recently established where children are presented with a Western arithmetic curriculum. In order to determine the way in which children interface their indigenous numeration system with the new press of "Western" arithmetic, a two phase study was conducted.

During the first phase, Grade 2, 4, and 6 children were observed as they solved arithmetic problems in class. With increasing grade level, children showed decreasing frequency with which they overtly used conventional body parts to help solve the arithmetic problems.

During the second phase, the same children as well as non-schooled controls were interviewed about their overt as well as covert solution strategies to arithmetic problems. Analysis of the interviews indicated that with increasing grade level, children employ increasingly adequate body part computational strategies to solve the problems. The nature and the significance of these strategies are discussed.

SUBMITTER'S NAME: Geoffrey B. Saxe

SUBMITTER'S SOCIAL SECURITY NUMBER: 555-78-1273

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Social Change and Cognitive Growth: The Invention of Body Part
Arithmetic Algorithms Among Oksapmin Children of Papua New Guinea

555-78-1273

The purpose of this paper is to analyze new arithmetic algorithms that are emerging among Oksapmin children in a remote area of Papua New Guinea. The major thesis is that "primitive" forms of numerical thought change to more abstract forms as a result of individual's cognitive adaptations of new problems, problems for which the primitive forms are ill-suited. The new adaptations are not a simple replacement of old forms of numerical thought with new ones; rather, the change in numerical thought is reflected in progressively more efficient means of employing the traditional forms to solve new problems. In the present paper, the concern is to analyze Oksapmin children's development of novel computational strategies as a result of new cultural press—their experience of a Western arithmetic curriculum presented in a Western styled elementary school.

The Oksapmin counting system consists of naming and/or pointing to 27 body parts beginning from the thumb on one hand, following the upper periphery of the body, and ending at the little finger of the opposite hand (see Figure 1). There is little, if any, social context in the traditional community to engage in arithmetic computations; however, in 1973 a bush elementary school was established by the government, and, as a part of school activities, children are presented with a Western arithmetic curriculum. In order to determine the way in which children interface their indigenous knowledge system with "Western" arithmetic, a two phase study was conducted.

Phase 1

In order to determine whether Oksapmin children used the indigenous system in school arithmetic, observers coded whether Grade 2 (25 Ss), Grade 4 (25Ss), and Grade 6 (22 Ss) children were pointing around their bodies during an arithmetic test. The frequencies with which children demonstrated an overt use of conventional body parts by grade level is presented in Table 1. With increasing grade level, children's overt use of their bodies declined ($\chi^2_{(2)} = 14.92, p < .01$).

Phase 2

All children who participated in Phase 1 were interviewed individually to determine whether they know ways to use the indigenous system to solve arithmetic problems. In addition, a group of non-schooled children who were approximately matched for age with the grade 6 children were also interviewed to determine the effects of the schooling experience. Each interview consisted of presenting children with four arithmetic problems orally, one by one ($5 - 3$ pigs, $3 + 5$ pigs, $14 + 7$ pigs, $16 - 7$ pigs). Children were asked probe questions about whether they knew how to use their bodies to help them solve the problems. If children did demonstrate "adequate" body part counting strategies (defined below), they were presented with additional arithmetic problems in order to determine the limits of their ability to use the indigenous system to facilitate their problem solving ($12 + 34$, $9 + 43$, $41 - 6$, $43 - 7$).

For each set of problems, adequate body part counting strategies were considered those which incorporated a "stop rule," a procedure whereby children could determine when the correct sum was achieved while counting around their bodies. Typically these rules involved establishing correspondence relations between body parts such that one body part was given the name of another body part. For example, to add $14 + 7$, a child might begin with the nose (14), call the eye the thumb (15 = 1), the ear the first finger (16 = 2), and continue on until the forearm of one side of

the body was called the forearm of the other side ($21 = 7$) and thus achieve the correct answer (see Figure 1). Adequate procedures for the second set of problems took a similar form and required that children count in English (by Grade 2 all children count in English). For example, to add $12 + 34$, a subject would call thumb 35, first finger 36, second finger 37, ... ear 46 (see Figure 1). In contrast to the adequate strategies, the inadequate strategies did not employ a "stop rule". These typically consisted of successively iterating body parts by their actual names until the subject stopped, presumably estimating the appropriate stopping place.

Tables 2 and 3 contain the frequency distributions of children's use of adequate and inadequate body part counting strategies for two sample problems from each series, $14 + 7$ and $16 - 7$ (Table 2) and $9 + 43$ and $41 - 6$ (Table 3). These Tables show that the 4th and 6th graders achieve adequate stop rules for the within range problems more frequently than the 2nd graders ($14 + 7$: $\chi^2_{(1)} = 16.29$, $p < .01$; $16 - 7$: $\chi^2_{(1)} = 21.1$, $p < .01$) and that the 6th graders achieve adequate stop rules for the second series of problems more frequently than the 4th and 2nd graders ($9 + 43$: $\chi^2_{(1)} = 4.97$, $p < .05$; $41 - 6$: $\chi^2_{(1)} = 6.70$, $p < .01$). Moreover, the non-schooled children show little evidence relative to the 6th graders of how to invoke a "stop rule" to solve even the first set of problems ($14 + 7$: $\chi^2_{(1)} = 21.6$, $p < .01$; $16 - 7$: $\chi^2_{(1)} = 23.65$, $p < .01$).

In sum, children invent algorithmic procedures based upon the indigenous system to more adequately cope with a new cultural press—the experience of a Western arithmetic curriculum. Moreover, these body part strategies undergo a significant intellectual development over increasing grade level, although with increasing grade level, they tend to assume a covert form in problem solving activity.



Table 1
Frequency with Which Children Used Conventional Body
Parts During Arithmetic Test by Grade Level

<u>Grade Level</u>	<u>Conventional Body Parts Used</u>	<u>Conventional Parts Not Used</u>
Grade 2	15	9
Grade 4	13	12
Grade 6	2	20

Table 2
Frequency of Use of Adequate and Inadequate Body Part Computational
Strategies for Two Series I Problems by Grade Level

<u>Grade Level</u>	<u>Arithmetic Problem and Strategy Type</u>					
	<u>14 + 7</u>			<u>16 - 7</u>		
	<u>Inadequate Body Strategy</u>	<u>Adequate Body Strategy</u>	<u>No Body Strategy</u>	<u>Inadequate Body Strategy</u>	<u>Adequate Body Strategy</u>	<u>No Body Strategy</u>
Grade 2	14	5	(4)	16	6	(1)
Grade 4	4	14	(6)	3	16	(5)
Grade 6	2	14	(6)	2	16	(5)
Non-Schooled Ss	18	2	(0)	18	2	(0)

Table 3
Frequency of Use of Adequate and Inadequate Body Part Computational
Strategies for Two Series II Problems by Grade Level

<u>Grade Level</u>	<u>Arithmetic Problem and Strategy Type</u>					
	<u>9 + 43</u>			<u>41 - 6</u>		
	<u>Inadequate Body Strategy</u>	<u>Adequate Body Strategy</u>	<u>No Body Strategy</u>	<u>Inadequate Body Strategy</u>	<u>Adequate Body Strategy</u>	<u>No Body Strategy</u>
Grade 2	4	3	(0)	5	2	(0)
Grade 4	9	7	(0)	11	5	(0)
Grade 6	3	12	(0)	4	11	(0)

4. Body parts as a standard measure: The development of measurement operations among the Oksapmin of Papua New Guinea

To be submitted at a later date

Section VI.

New Directions for Number Development Research

NEW RESEARCH DIRECTIONS:

A Developmental Analysis of Numeration Concepts:
The Effects of Language Background, Mother-Child Interaction,
and Metacognitive Processes

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DESCRIPTION AND RATIONALE

Overview

It is well known that certain social factors such as ethnicity, language background, and social class are associated with low levels of educational opportunity and scholastic achievement among children (e.g., Coleman, 1966). The inequities in educational opportunities between middle class and underprivileged groups can be related to two general types of factors. The first are concerned with out of school experiences which influence the preparedness of children to enter formal school settings (e.g., Bee, Van Egaren, Straissguth, Nyman, & Leckie, 1969; Brophy, 1970; Hess & Shipman, 1965, 1967; Zigler, Abelson, & Seitz, 1973; Zigler & Butterfield, 1968). The second are concerned with in-school experiences and may involve across group differences in teacher/student ratios, the committedness of teachers and administrators to quality education, and the general educational atmosphere of the school (Edmonds, 1979). These two general sets of factors—those associated with in- and out of school learning environments—undoubtedly interact. The child who comes to school ill prepared, lacking confidence, and disinterested in school learning is unlikely to engage the teacher's interest in imparting knowledge, just as a teacher who lacks concern and functions in a setting which is not committed to quality education is unlikely to engage the child in learning opportunities.

Policies which will lead to effective intervention in this cycle which perpetuates inequitable opportunities for the poor and minority populations must come from an increased understanding of how both sets of factors arise, and thus how each can be altered. The planned research focuses on the first

factor--sources of different levels of preparedness among children of different social groups to enter and benefit from the school learning environment. If we can determine some of the general strengths as well as weaknesses of the out of school learning environments of poor and minority populations, we will be in a better position to adjust educational curricula, both to meet these particular children's needs and to draw out and encourage particular areas of competence.

The planned project will follow up and extend three areas of research on mathematics learning for which we have already completed exploratory work. The first pertains to the effect of language background on the development of mathematical cognition. We plan to extend our previous findings that middle class bilingual children show a distinct advantage over their monolingual peers in certain aspects of mathematics learning involving numerational concepts and not others, such as conservation concepts (Saxe, Schonfeld, Sicilian, & Segal, 1979; Saxe, Sicilian & Schonfeld, Note 1--Appendix A) to minority populations who live in the inner city. If we find that among these inner city populations such an effect for language background occurs as well, it would suggest curricula for bilingual education programs draw upon these childrens' particular areas of strengths.

The second area of research concerns early learning and problem solving in the home environment. We plan to extend our previous research on children's counting in problem solving activities (Saxe, 1977; Saxe, 1979; Saxe, Cohen, & Kaplan, Note 2) to the way in which these activities are nurtured and facilitated in mother-child interactions. We suspect that differences in the nature of these mother-child interactions may be an early source of differences in preparedness for school mathematics among children. If differences in

styles of interaction can be identified which lead to different degrees of preparedness in mathematics learning, it will inform programs for compensatory education as well as parent involvement programs.

A third area concerns a possible source of children's failure in the acquisition of basic arithmetic concepts. Various authors have noted the heuristic value of "metacognitive skills"—or the ability to monitor one's own cognitive activity—across various areas of problem solving (e.g., Brown, 1978; Flavell, 1979; Flavell & Wellman, 1977; Markman, 1977) although there has been virtually no research on the development and use of metacognitive skills in the context of counting activities and early arithmetic problem solving. This area of planned research focuses on developmental changes in children's ability to estimate or monitor their own counting accuracy, and how this process is related to the child's acquisition of basic arithmetic concepts.

Introduction

The proposal is divided into three sections. In the first, the mainstream of past research on the development of numerical cognition is briefly reviewed. In general, this literature concerns children's developing understanding of number conservation and tends to neglect children's use and understanding of numeration concepts. It is argued that while developmental changes in children's understanding of conservation may be of interest in its own right, conservation is only indirectly related to fundamental problems which children experience in the acquisition of school related numeration and arithmetic concepts. The second section concerns children's counting and focuses on a recent body of research concerned with developmental changes in children's use of counting as a notational system for number. It is argued that the pursuit of research in this area is a potentially profitable contribution to the improvement of education in mathematics. Each of the three areas of planned research is then discussed--bilingualism, parent/child interaction, and estimation skills. It is argued that by increasing our knowledge about how each of these variables affects children's mathematics learning, we should be in a better position to make intelligent decisions about educational policy and curricula which would lead to greater equity in educational opportunity among children of different social groups. In the third section, the research design and procedures are presented for each of the three areas of the planned study.

Section I: Past Research on Number Conservation Concepts

In a pioneering set of studies on numerical cognition, Piaget (1952) argued that children's understanding of numerical relations undergoes a qualitative change in development. The young child conceptually confuses numerical and spatial relations, whereas the older child clearly differentiates between these relations. One of Piaget's classical studies on number conservation provides a good illustration of this developmental process. Children were presented with a linear array of elements and asked to reproduce the number with an available set. Piaget reported the following three-stage sequence: At the first stage (about 4 years of age), children simply matched the end-points of the arrays and not the actual number. At the second stage, children matched the elements one-for-one, but if one of the arrays was spread apart, children believed that the numerical relation between the two arrays also changed. It was not until the third stage (about 7 years of age) that children both accurately reproduced the number and understood that number was conserved through the spatial transformation of the elements, that is, that the one-to-one correspondence that the child initially produced was lasting.

Although there have been some researchers who have questioned the validity of Piaget's stage sequence (e.g., Gelman, 1969; Mehler & Bever, 1967; Bever, Mehler, & Epstein, 1968), for the most part, researchers have replicated Piaget's classical findings. The replications include studies with normal Western children (e.g., Almy, Chittenden, & Miller, 1966; Beard, 1963; Beilin, 1968; Elkind, 1964; Hood, 1962), non-Western children (e.g., see Dasen, 1974 for a review), learning disabled and mentally

retarded children (e.g., Boland, 1973; Inhelder, 1943; McManis, 1969; Saxe, & Shaheen, in press) and economically disadvantaged children (e.g., Ginsberg & Russell, Note 3). Numerous researchers have also found that over a variety of task contexts, the young child has difficulty distinguishing between spatial and numerical properties of sets (e.g., Brainerd, 1977, 1978; Gelman, 1972; La Pointe & O'Donnell, 1974; Siegel, 1974), and that this differentiation can be observed in younger children when set size is reduced (e.g., Cowan, 1979; Silverman, Rose & Phillis, 1979; Winer, 1974; Young & McPherson, 1976; Bullock & Gelman, 1977; Gelman & Gallistel, 1978; Gelman & Tucker, 1974; Saxe & Cohen, Note 4), when non-verbal interview procedures are used (e.g., Miller, 1976), or when only one as opposed to two arrays are used in a conservation task (see Brainerd & Hooper, 1975 for a review).

Although children's developing understanding of number conservation is of interest in its own right, this process may be only indirectly related to fundamental problems which they experience in their acquisition of school related arithmetic concepts and computational skills (Easley, 1973; Groen & Resnick, 1977; Mpiagu & Gentile, 1975; Saxe, 1979a, b). In the following section, research is reviewed in an area of mathematical learning which is more directly related to school related mathematical skills—children's developing use and understanding of numerational concepts.

Section II: Research on Counting and Numeration Concepts

The bulk of past research on the acquisition of counting largely concerned age-related improvements in knowledge of the rote system and counting accuracy. In general, this literature indicated that these abilities improve between 2 and 5 years of age (see Gelman, 1972 for a review of this literature).

More recently there have been studies which focus on developmental changes in the counting strategies which children employ in solving tasks, and an analysis of the logical capacities which underlie children's counting behavior (e.g., Beilin, 1975; Fuson, Note 5; Gelman & Gallistel, 1978; Ginsburg, 1977; Resnick, 1979; Russac, 1978; Saxe, 1977, 1979a, b, c; Schaeffer, Eggleston & Scott, 1975; Shannon, 1979; Steffe, 1978). In general, this literature documents developmental changes in the way in which children use counting as a symbolic vehicle to represent numerical relations. Further review of recent research on counting and numeration will be presented under the three headings below—language background, mother-child interaction, and metacognitive processes. Under each heading, I attempt to show how the extension of research on counting and numeration to each area of study will provide a useful and important contribution to our knowledge about mathematics learning.

1. Language background: The value of bilingualism in mathematics learning.

Whether bilingual children are advantaged or handicapped relative to their monolingual peers in their understanding of mathematical concepts is a complex empirical issue with significant implications for issues regarding bilingual and bicultural education. There have been some researchers who have claimed that bilingualism interferes with general cognitive development (cf. Arsenian, 1937; Peal & Lambert, 1963 for reviews), others who have claimed that it may have no effect but may have positive effects upon personal-social development (e.g., Bruck, Lambert, & Tucker, 1974; Lambert & MacNamez, 1969; Lambert & Tucker, 1972), and others who have claimed that it facilitates cognitive growth and flexibility (e.g., Balkan, 1972; Ben-Zeev, 1977; Ianco-Worrall, 1972; Liedtke & Nelson, 1968;

Peal & Lambert, 1962). The most critical factor which contributes to these contradictory positions and findings is the general lack of a coherent theoretical formulation which specifies how bilingualism interacts with specific domains of intellectual functioning. The measurement instruments which researchers have chosen to assess intellectual development reflect this lack of formulation. For instance, to assess differential effects of bilingualism on mathematics learning, studies have typically employed a global procedure for assessing mathematics competence in the form of one or another pre-packaged standardized achievement test of arithmetic achievement (e.g., the Metropolitan). An analysis of the relation between bilingualism and mathematics of any depth would require theoretically motivated measurements of each. In addition to the problem of both the absence of a clear theoretical rationale and appropriate measurement instruments, investigations have often confounded two other factors associated with the subject populations assessed. One factor is relative competence in each language; studies have pooled coordinate bilinguals—those who can use two languages interchangeably—with other types of bilinguals. The other factor is social class; studies have confounded social class with bilingual-monolingual contrasts (cf. Peal & Lambert, 1963 for a review). As a result, the nature of the relation between bilingualism and mathematics learning has been obscured.

In this section of the proposal, a theoretical rationale for an effect of language background on mathematics learning is presented, together with preliminary findings which support it. In the Design and Procedures section of the proposal, particular measures and subject populations are specified which will provide empirical tests of this framework.

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As part of a general theoretical formulation of the development of numeration, we have argued that it is important to distinguish between two aspects of mathematical understanding (Saxe, 1979a, b; Saxe, Schonfeld, Segal, & Sicilian, 1979; Saxe, Sicilian, & Schonfeld, Note 1). One concerns "numerational concepts" and pertains to an understanding of the structure of numerational systems as symbol systems for number. The other concerns "number concepts" and concerns the understanding of numerical operations (e.g., one-to-one correspondences, addition and subtraction). We have argued that it is within the domain of numerational concepts that bilingualism should interact with the development of mathematical understanding.

The argument that bilinguals should show an advantage over their monolingual peers in understanding numeration stems from a consideration of the structure of numerational concepts themselves. Numerational concepts involve the understanding that symbols for number have both conventional and logical properties. For instance, the particular symbols that we use to represent number, "1", "2", "3", ..., are conventions and themselves an arbitrary feature of the activity of counting. However, the activity of establishing the one-to-one correspondences between number names and a target set of elements is a logical property of any act of counting. In a recent series of studies, we began to investigate children's understanding of the formal organization of the numerational system, that is, that it has both logical and conventional properties. We had two major hypotheses. The first was that children would progress from a phase in which they did not differentiate between conventional and logical properties of numeration to a phase in which they did understand this differentiation. The second was that, since bilingual children have direct experience with multiple systems of signification for the same

referents, they would achieve this differentiation prior to their monolingual peers, even though these groups may not differ with respect to understanding of basic "number" concepts which do not entail this form of conceptual distinction.

As a means of testing these hypotheses, we developed a paradigm in which we examined children's ability to understand aspects of the formal organization of our numerational system. The paradigm consisted of telling a child a story which involved the use of two finger puppets—Bert and Ernie from the television show Sesame Street. A child was told that Bert was really named Bert-A and came from a country far away where people use the alphabet to count, and that Ernie was really named Ernie-1 and came from New York where people use numbers to count. In our story, the two finger puppets met and went to the supermarket where they counted pictures of fruit. When Bert-A counted, he used the alphabet and when Ernie-1 counted, he used number names. After each count, they argued about who counted correctly and the child was asked to intervene and to tell who counted correctly and to explain his or her answer.

Using this paradigm, a series of studies was conducted across a wide range of conditions and a variety of subject populations (see appended manuscript). As predicted, we found that over 4-, 7-, and 11-years of age, children progressed from an understanding that Bert-A is an inadequate counter to a understanding that he is an adequate counter. Moreover, also as predicted, we found that, of a population of first graders who were administered these procedures as well as number conservation procedures, bilingual children performed significantly better than their monolingual peers on the Bert and Ernie conditions, but not on the number conservation conditions.

These findings on the child's understanding of numeration need to be followed up in two ways. The first concerns the generality of the findings across social class. Whether the previous finding for middleclass children of an effect for language background exists for lower SES children is clearly an important issue for bilingual education. The second issue concerns the generality of the findings on conventional-logical properties of numeration to other types of numeration concepts, for example, base structure. The understanding that numerational systems may have different base structures is an important concept in the technological world. For instance, although the common form of representing "ten" elements is the written symbol "10", in a base-two numerational system—those used in digital computers—"ten" elements is written as "1010". Similar forms of understanding underlie translations between different systems of measurement (e.g., understanding the translation between measurement systems used in the United States and the Metric system). The cognitive underpinnings of children's numerational concepts and the effect of language background on children's development of these concepts is yet to be explored.

2. The development of numeration abilities: The role of mother-child interaction.

Mother-child interaction as a form of cognitive socialization is an area of considerable interest, both from the points of view of developmental theory as well as educational practice. From a theoretical point of view, it provides a domain of analysis in which the cognitive achievements of culture are transmitted to the growing child. From an educational point of view, there may be more and less adequate ways of achieving this process of transmission.

Although the study of cognitive socialization to date has not been extended to mathematics learning, such an extension should be of considerable value. In this section, some of the previous thinking and research about cognitive socialization is briefly reviewed. It is argued that the analyses of both cognitive socialization and mathematics learning would benefit from their integration into a single program of research.

Various researchers have studied the differences between middle and lower SES mothers in the way in which mothers interact with their children when the mothers are instructed to teach the child a task (Bee et al., 1969; Brophy, 1970; Hess & Shipman, 1965), or when they interact with their child in an unstructured problem solving setting (Bee, et al., 1969). Brophy found that, in general, all mothers structure the task for the child; however, during each phase of this process, there were differences between middle and low SES mothers. Middle SES mothers tended to prepare their children for the task more adequately and focus their child's attention to a greater extent. Hess & Shipman (1965), in an earlier study, found that lower SES mothers tended to criticize more and give less praise than middle class mothers. In a similar vein, Bee et al (1969) found that lower SES mothers were more controlling, more disapproving, and were less informative to their children than middle class mothers.

Whether these differences in "cognitive socializing practices" have significant cognitive developmental consequences is not altogether clear. Ginsburg (1972) has pointed out that often the differences were small between the groups and that the problem solving tasks were artificial and not the typical kind of task which a mother might be teaching to children in a home

setting. Moreover, a general problem with this research is that it has lacked a clear theoretical formulation about how cognitive socialization works as a factor which interweaves with the cognitive development of the child. Without such a formulation, it is somewhat arbitrary to isolate certain domains of behavior for analysis and not others, and, without a theory to guide, differences in interactions across social class are difficult to interpret. Recently, Vygotsky's general theoretical formulation has enjoyed a renewed interest insofar as it can serve as a general framework to understand the significance of social interaction from the point of view of cognitive development (cf. Cole & Scribner, 1974). Below, I briefly review Vygotsky's approach as well as recent empirical work which has followed from his approach.

Vygotsky (1962, 1973) has outlined a formulation of cognitive development in which knowledge that is a product of social history is acquired in social interaction with adults. According to Vygotsky, this occurs through a process whereby adults engage children in activities which children are not completely capable of doing on their own but that are within the grasp of their understanding—activities which are within the child's "zone of proximal development." By guiding children through cognitive tasks, adults introduce particular problem solving strategies and symbols to children. Although at first these strategies and symbols are regulated by the adult in the problem solving situation, over the course of development the child progressively internalizes these symbols and strategies and comes to use them successfully on his own. Thus, through this process, Vygotsky claimed, children acquire the capacity to regulate their own problem solving behavior and at the same

time interiorize the cognitive achievements of culture.

In an attempt to follow up Vygotsky's approach to cognitive development empirically, Wertsch (1979) studied mothers and their 2-, 3-, and 4-year-old children engaged in solving a simple puzzle. Wertsch showed each mother/child dyad a model puzzle and an intact copy. He then took the copy apart. The mother was told that the child was supposed to make the copy look exactly like the model. The mothers were instructed simply to help their children whenever they thought that it was necessary. The sessions were videotaped, and the videotapes were transcribed and coded for various behaviors such as pointing, gazing, and handling pieces in order to analyze the strategies that the mothers used to regulate the child's behavior. Consistent with Vygotsky's formulation, Wertsch found that the form of dyadic interaction changed over age in his sample. In the case of young children, the mothers played a major role in regulating the child's problem solving activities. With increasing age, the child's problem solving became increasingly self-regulative. Wertsch argues that his data support a developmental sequence composed of four levels that correspond to the gradual transfer from other- to self-regulated problem solving behavior.

Although Wertsch's work is important insofar as it documents some important changing behavior patterns in mother-child interaction, there are several areas in which it is weak. Most significantly, Wertsch did not have a developmental analysis of children's puzzle solving behavior. Without such an analysis it is not possible to identify the way in which the mother's interventions and activity mesh with the child's own abilities and understandings (Wertsch had no pre-test of children's behavior). Hence, differences in the styles of interaction between mother and child as a function of the child's

own ability level must have been obscured. In addition, Wertsch's research did not include a follow-up study. Since the Vygotskian formulation is one which is particularly geared towards an analysis of the influence of culture on developmental changes in cognition, it would make sense to follow mother-child pairs over time to determine both the ways in which individual styles of interaction change over time as well as whether some styles are associated with more rapid or greater "depth" of understanding than others.

In the planned research which is described in the Design and Procedures section, we will address some of the major weaknesses in previous work on cognitive socialization. Since we have already produced a developmental analysis of children's use of counting (Saxe, 1977, 1979b, Saxe & Cohen, Note 4), and have developed a coding scheme for children's counting behavior (Saxe, Cohen, & Kapan, Note 2, Appendix 3), we will be able to pre-test children as individuals, assess mother-child interaction in a learning setting, as well as follow-up these same mother-child pairs longitudinally. The planned research should also provide a basic contribution to normative work on the development of counting and numeration. As noted previously, research on the early emergence of number symbolization has not concerned the process whereby children acquire counting and numeration concepts in social interaction with adults (although see Saxe, 1979c; in press; Note 6, for some considerations of this problem from a cross-cultural perspective). An analysis of how the process of cognitive socialization occurs for early numeration, the types of similarities and differences which occur in this process over mother-child dyads, the consequences of these differences for the acquisition process, are all important next steps for research on children's early mathematics learning.

3. Metacognitive processes: Developmental changes in children's ability to monitor their counting activities.

Children's metacognitive skills, or their ability to monitor their own problem solving and cognitive activities, is becoming a focus of interest among researchers in cognitive development (see Flavell, 1979 for a review and programmatic statement). For the most part, investigations of metacognition have been concerned with issues related to memory (Brown, 1978; Flavell & Wellman, 1977) and language (cf. Sinclair, Jarvella, & Levelt, 1979) and generally have not addressed the study of mathematics learning. In the model presented below, two aspects of children's counting are distinguished—structural and metacognitive aspects. To date, the distinction between structural and metacognitive aspects of counting activity has not been made explicit in studies of counting behavior, although various researchers have recognized the need for this type of distinction (cf. Gelman, 1972; Gelman & Gallistel, 1978; Saxa, 1977; 1979a; Shannon, 1978). The interrelations between these two aspects are discussed below, and appropriate next steps for research on developmental changes in children's metacognitive skills with respect to structural aspects of counting are presented.

The structural aspect of counting entails knowing that, in principle, the counting system can be used to represent number; that is, by assigning a series of number names to a target set of elements in one-to-one correspondence, each successive number name can be used to represent a summation of one-to-one correspondence relations. The metacognitive aspect of counting entails understanding that, in any particular count, one can be confident

or not so confident that, in fact, one has successfully produced a one-to-one correspondence between the series of number names and objects. This aspect may be manifested both in strategic behaviors children use to maximize the possibility of an accurate correspondence between number names and elements (e.g., moving objects aside as they are counted), and also in children's reflections on the probability of counting errors in situations in which there are constraints upon strategic behaviors which they can deploy (e.g., constraints such as counting sets that are inside a jar as opposed to counting sets which can be touched and moved). Research is planned on both strategic and reflective metacognition.

With regard to strategic metacognition, there is one form of seemingly strategic behavior—the pointing gesture—that is used in counting by very young children (cf. Gelman & Gallistel, 1978; Saxe, 1977; 1979b), but whether it is indeed used strategically by the young child to effect an accurate count is a matter of some controversy. On the one hand, Gelman & Gallistel (1978) have argued that the very young child evidences a structural understanding that number names must be assigned in one-to-one correspondence to elements and that the young child invokes pointing gestures (a strategic behavior) to serve this numerical function. In contrast, Saxe (1979b) has argued that the structural competence underlying children's use of number names undergoes developmental changes, and therefore that the strategic function of these pointing gestures relative to counting-like and counting activities must change as well. On this view, pointing gestures are at first used as simply a means to refer number names to target elements, and it is only over the course of development that the function of pointing gestures changes such that the child uses them strategically to mediate an accurate

count. In the planned studies, an experiment is presented that will be used to test the relative adequacy of these models.

With regard to reflective metacognition, little is known about developmental changes in children's ability to interpret or reflect upon their own counting accuracy. For instance, there are situations in which one counts an array and is not sure whether an accurate count was produced. Whether children monitor their counting errors, know the consequences of their errors on their final sum, know the circumstances which will lead to greater or fewer errors in counting are all unknowns at this point, as are the experiences which may lead to the development of these metacognitive capacities. The planned research addresses each of these issues.

A critical question for all researchers interested in the development of metacognitive skills--but one which the planned research does not address directly--is whether and in what way metacognitive skills are related to more basic forms of structural understandings. It is likely that children's ability to monitor their counting accuracy is related to the learning of basic arithmetic and computational skills. If children miscount and do not fully appreciate the consequences of their miscounting as they experiment with addition and subtraction operations on sets, they are likely to have difficulty achieving an understanding of arithmetic operations. The proposed research takes an important step in documenting aspects of the development of metacognitive skills in the use of numeration. In future studies, it is expected that we will extend the research outlined in this proposal to an analysis of the influence of metacognitive skills on the development of arithmetic operations.

Section III: Design and Procedures

In this section, planned studies are outlined that will extend our knowledge in each of the three areas previously reviewed.

1. Language Background: The value of bilingualism in mathematics learning

The studies presented in this section extend our research with middle class bilingual children to minority inner city populations (Hispanic and Chinese) and to forms of numerational concepts which we have not previously studied. The general strategy is to document developmental sequences in children's acquisition of various forms of numerational concepts and to determine whether lower SES bilingual children show special strengths over their monolingual peers in their understanding of these concepts. In addition to the numerational concepts under investigation, in each of the studies in which we contrast bilinguals with monolinguals, children will be assessed on a non-numerational concept, number conservation. We expect that in all cases, bilinguals will show an advantage over monolinguals on the numeration tasks; however, we expect that there will be no difference on their levels of understanding conservation. (In effect, we expect that monolinguals and bilinguals will be matched on their understanding of conservation.)

Study 1

In this study we make use of the same paradigm we used in previous research on children's understanding that the activity of counting has conventional and logical properties (Saxe, Schonfeld, Sicilian, & Segal, 1979; Saxe, Sicilian, & Schonfeld, Note 1). As in the previous procedure, the child is asked to resolve a dispute between two puppets, Bert-A who comes

from a country where the alphabet is used in counting, and Ernie-1 who comes from New York where number names are used in counting.

Subjects. A sample of 90 children who attend inner city first grade classes in New York and who are of different language backgrounds will be interviewed (30 Spanish/English speaking bilinguals, 30 Chinese/English speaking bilinguals and 30 English speaking monolinguals). Each group will contain 15 boys and 15 girls. Language background will be determined in conference with teachers and the school administration.

Procedures. The experimenter will interview each child individually. The counting and the conservation conditions outlined below will be presented in a counterbalanced order. Prior to the administration of the procedures, we will screen subjects on their abilities to count and recite the alphabet by rote. (cf. Appendix A, p. 6 for details.)

Introduction to counting conditions. The child is presented with Bert and Ernie finger puppets and told "This is Ernie-1 and this is Bert-A (the interviewer turns the puppets around and shows the child their labels). Ernie-1 comes from down the street where they use numbers to count, but Bert-A comes from another country far away where they use letters to count." The interviewer questions the child in order to determine if the child understands. If the child does not understand this introduction, it is repeated and the interviewer checks the child's comprehension again. (cf. Appendix A, p. 6.)

Counting conditions: set size three. (a) No violation condition. The interviewer tells the child the following: "One day Ernie-1 and Bert-A were walking along. They saw a store and went in." The interviewer presents the picture of three apples, and asks the child to identify them. (Ernie-1 then says:) I WONDER HOW MANY THERE ARE. I'LL COUNT. THERE ARE 1, 2, 3 (counts

with his nose). THERE ARE 3. (Bert-A says:) NO, THAT'S NOT RIGHT, THAT'S NOT RIGHT AT ALL. THERE ARE A, B, C. (counts with his nose). THERE ARE C. The interviewer then asks the child, "What do you think? Did both Bert-A and Ernie-1 count right, or did one count right and the other count wrong?" After the child responds, the interviewer asks, "How do you know?"

(b) Numeral repetition condition. The interviewer tells the child, "Ernie-1 and Bert-A walked further on in the store." The interviewer presents a picture of 3 oranges, and asks the child to identify them. (Ernie-1 then says:) I WONDER HOW MANY THERE ARE? I'M GOING TO COUNT THEM. THERE ARE 1, 1, 2. THERE ARE TWO. (Bert-A says:) NO, THAT'S NOT RIGHT, THAT'S NOT RIGHT AT ALL. THERE ARE A, B, C. THERE ARE C. (The interviewer then asks:) "What do you think? Did both Bert-A and Ernie-1 count right, or did one count right and the other count wrong?" After the child responds, the interviewer asks, "How do you know?" and additional probe questions.

Counting conditions: set size six. The two conditions administered with set size six are identical to the conditions administered with set size three. In the numeral repetition condition, Ernie-1 will repeat the numerals "2" and "3" twice each.

Number conservation condition. (1) One-to-one correspondence phase. The interviewer presents the child with a linear array of nine 2 cm diameter checkers and an available set of similar checkers. The interviewer tells the child to put out one checker for each one in the model row. The child's reproduction is recorded. If the child does not make an accurate reproduction, the interviewer helps the child. Immediately following this phase, the number conservation phase is administered. (2) Number conservation phase. Once the two arrays are aligned in one-to-one correspondence, the

interviewer says, "Now, do we both have the same number?" Once the child responds affirmatively, the interviewer says, "Now watch what I'm going to do." The modal array is then spread apart and the child is asked, "Do we both have the same number, or does one of us have a bigger number?" "How do you know?" This procedure is repeated with a compression transformation.

Data analysis. Children's judgements about the adequacy of the puppets' counting will be analyzed for each of the counting conditions (see appended manuscript for description of scoring criteria) and children's conservation responses will be assessed using both a judgments-only and a judgments + explanations criteria. Chi-square analyses will be used to determine (1) whether groups differed with respect to their understanding of conservation, (2) whether groups differed on the counting conditions, (3) whether there are within group differences across the counting conditions.

Study 2

The previous study was concerned with children's understanding that there can be alternate symbols for representing number or numerical relations (e.g., letters or numbers names). The focus of the present study is to investigate developmental changes in children's understanding that symbols for number take on values in reference to the particular rule system in which they occur. Put in another way, in this task we are concerned with children's understanding that numerational systems can be organized with different base structures, such that the same symbol in one system has a different meaning in another system (e.g., "10" is not the equivalent of "10" if one is used in a base-two and the other in a base-ten system).

In order to assess developmental changes in children's abilities to understand this type of relation between two different numerational systems, middle class children at three age levels will be presented with a new Bert and Ernie story. In the subsequent study (Study 3) the results will be followed up with inner city bilingual and monolingual children.

Subjects. A sample of sixty monolingual children from middle SES backgrounds who are 5-, 8-, or 11- years of age will be interviewed. Each age group will contain 10 boys and 10 girls.

Procedures. After a brief introduction, the child will be presented with a pre-test involving knowledge about making change with coins. In order to avoid problems involved with using real change, color photographs will be taken of coins, the coins will be cut out and affixed to plastic discs, and these materials rather than real change will be used in the session. In this session, children will be queried about their knowledge about equivalence relations between coins, e.g., 5 pennies = 1 nickel, 2 nickels = 1 dime, etc. Only subjects who demonstrate an initial competence with these relations or who can be trained to a criterion with ease will participate in the study.

In the story condition phase, the child is told that Bert and Ernie live in different countries (a large mat board is used with a border in the middle as a graphic layout—see Figure 1). In both countries people use pennies, nickels, and dimes for money (the play pennies, nickels, and dimes are displayed). In Ernie's country if he wants change for a nickel, he gets five pennies, and if he wants change for a dime, he gets two nickels. In Bert's country, however, if he wants change for a nickel, he gets two pennies, and if he wants change for a dime, he gets four nickels (these relations are represented graphically and the child will be able to refer to them).



Bert's country	Ernie's country



The child will be told a series of six episodes (randomized) three of which occur in Ernie's country and three in Bert's. The stories will take the following general form: One day Ernie went to Bert's country to visit. He just brought pennies and he wanted other change. Ernie then gives Bert some pennies and Bert gives Ernie some change. They argue about the change and the child is asked to intervene to mediate the dispute and explain what Ernie should get in return. Ernie's country, Conditions: (1) 4 pennies = 2 nickels, (2) 3 pennies = 1 nickel + 1 penny, (3) 5 pennies = 2 nickels + 1 penny. Bert's country, Conditions: (1) 5 pennies = 1 nickel; (2) 7 pennies = 1 nickel + 2 pennies, (3) 11 pennies = 2 nickels + 1 penny.

Data analysis. Each condition will be analyzed separately. Chi-square analyses will be used to determine whether there is an effect for age group and to determine whether there is an effect for condition types within each age group.

Study 3

The purpose of this study is to extend the findings of Study 2 to bilingual/monolingual comparisons.

Subjects. A sample of 90 children who attend inner city first grade classes in New York and are of different language backgrounds will be interviewed (30 Spanish/English speaking bilinguals, 30 Chinese/English speaking bilinguals, and 30 English speaking monolinguals). Each group will contain 15 boys and 15 girls. Language background will be determined in conference with teachers and the school administration.

Procedures. The experimenter will interview each child individually. The counting condition outlined in Study 2, and the conservation conditions outlined in Study 1 will be presented in a counterbalanced order.

Data analysis. Each condition will be analyzed separately. Chi-square analyses will be used to determine whether there is an effect for language group for each condition and to determine effects for condition types within each language group. As indicated in the introduction, we expect no effect for number conservation performance across language groups, but we do expect significant differences for bilingual/monolingual.

Study 4

The purpose of this study is to investigate children's understanding that the figural properties of numerational terms need not have any relation to the values which they represent.

Subjects. Sixty children will participate in this study (20 at 5-, 8-, and 11 years of age).

Procedures. The experimenter will interview each child individually. Children will be presented with Bert and Ernie puppets and told that each puppet lives in a different country (again using the mat board format). In both countries people use small blue and big yellow chips for money (small blue and big yellow plastic discs). In Bert's country, 1 big yellow

disc is worth 5 small blue ones. In Ernie's country, 1 little yellow disc is worth 5 blue ones (props are presented as reminders of these relations). In the story, one puppet goes to visit another puppet and needs change. Four conditions are presented in randomized order. In each condition, the two puppets argue about what is the right amount to receive in return. In each case, the child is asked to intervene, mediate the dispute and explain the correct solution.

Ernie's country, conditions: Bert gives 5 big blues,
Ernie gives 1 little yellow

Bert gives 1 little yellow,
Ernie gives 5 big blues

Bert's country, conditions: Ernie gives 1 big blue,
Bert gives 1 yellow

Ernie gives 5 little yellows,
Bert gives 1 big blue

Data analysis. Children's responses will be scored as to whether they understand that figural properties do not enter into the value of a chip. Chi-square analyses will be used to determine whether there is an effect for condition type at each age level, and whether children's understanding of the tasks differ over age level.

Study 5

The purpose of this study is to extend the finding of Study 4 to bilingual/monolingual comparisons.

Subjects. A sample of 90 children who attend inner city first grade classes in New York and are of different language backgrounds will be interviewed (30 Spanish/English speaking bilinguals, 30 Chinese/English speaking bilinguals, and 30 English speaking monolinguals). Each group will contain 15 boys and 15 girls. Language background will be ascertained

in conference with teachers and the school administration.

Procedures. The experimenter will interview each child individually. The counting conditions as outlined in Study 4 and the conservation conditions outlined in Study 1 will be presented in a counter balanced order.

Data analysis. Chi-square analyses will be used to determine whether there is an effect for language group and to determine effects for condition types within each language group.

2. The development of numeration abilities: The role of mother-child interaction

Study 6

The aim of the research in this area is to follow up research on both cognitive socialization and early mathematics learning. The study focuses on mother-child interaction in the context of counting activities. The specific goals of the planned research are to determine (1) the ways in which children's ability levels influence the kinds of inputs mothers offer children and the ways in which mother's inputs affect children's learning, (2) whether the styles of mother-child interactions in the context of early mathematics learning differ over social class, (3) whether some styles lead to greater mastery (as assessed in a one year follow-up), (4) whether mother-child interactions change in these activities over age of child along the same lines as other investigators have reported for other contexts (e.g., Wertsch, 1979).

Subjects. Subjects will include 100 3-year-olds (50 middle SES and 50 low SES) and their mothers. The mother-child dyads will be retested in a one-year follow-up study. We expect about a 50% recovery of our initial sample in the 1 year follow up. We will obtain mother-child dyads through contacts at day care centers and pre-schools in the New York City area.

Procedures. The study will be conducted in a series of two phases, an initial assessment and a one-year follow-up. During each phase, children will be pre-tested with mothers absent on counting tasks, and then mother-child interaction will be observed as mothers work with their children on counting activities.

Consultation and warm-up. Prior to coming to the Graduate Center, a

research assistant will talk to mothers about the study. During this initial introduction, mothers will be informed that the study is about children's counting and about how children learn to count. After getting re-acquainted at the Graduate Center, we will show her and her child some of the materials that will be used in the study (described below) and point out some of the features of the materials (e.g., some are single arrays of toys, some are double arrays, some are loose toys — the materials are described below). The mother will then be interviewed by a research assistant while a second experimenter conducts a pre-test. After the pre-test is terminated, the interaction phase will begin. The instructions to the mother will be that we are interested in how children learn about counting in their homes, and could she show us how her child learns about counting with our toys.

Pre-test task conditions. The task conditions presented below will be used in the pre-testing of individual children.

Notational counting.

(a) Fixed materials. Cards with small sets (2 or 3 elements), intermediate sets (6 or 7 elements) and large sets (18 or 19 elements) of toy animals (homogeneous for each array) will be used in this task. The child will be required to count the toys.

(b) Cardinality. After each trial presented above, the array will be screened and children will be asked how many animals they counted.

(c) Movable stimuli. As above, but the stimuli will not be fixed to a card.

(d) Comparison tasks. Two rows of toy animals will be fixed to a card. Rows contain different numbers, but the spatial extents of the two rows will be matched. Three set size conditions will be used, small (3/4 elements), intermediate (8/9 elements), and large (18/19 elements). The child will

be asked if both rows have the same number, or if one has a bigger number.

Mother-interaction phase. Mothers will be asked to work with their children with the materials provided and to show how they teach their children about counting.

Data analysis. Children's pre-test performances will be analyzed with the coding scheme used in previous research (Saxe, 1977; Saxe, Cohen, & Kaplan, Appendix B).

A coding scheme for mother-child interaction will be constructed on the basis of the video tape data. The scheme will incorporate our own coding scheme for children's counting behavior (Appendix B), and aspects of Wertsch's (1979), Bee et. al.'s (1969), Brophy's (1970) and Hess & Shipman's (1965) schemes.

The codes developed will be used to analyze the videotapes. The final data analysis will be dependant upon the coding scheme, however, there are many areas of analysis anticipated. General questions to be answered include: Does mother-child interaction differ over social class if children are matched for pre-test performance? Does mother-child interaction differ within social class over differences in child's pre-test performance? Is the character of the mother-child interaction at session I predictive of the child's pre-test performance in the follow-up phase? In what way does mother-child interaction change over the initial and follow-up phases, and how are these changes related to the child's pre-test behaviors?

3. Estimation and metacognitive processes: Children's knowledge of their counting errors.

The studies in this section are each concerned with children's abilities to monitor and interpret their own counting accuracy. Study 7 concerns changes in children's use of pointing gestures to mediate their counting accuracy. Study 8 is concerned with developmental changes in children's ability to know when they have and the degree to which they have miscounted. Study 9 is concerned with the conditions under which children's ability to interpret their own counting accuracy improves.

Study 7

In this study the concern is to determine whether children use pointing gestures to mediate their counting accuracy, and, if not, when this ability begins to emerge.

Subjects. The sample will include 45 children, 15 at each of the three age levels (2½-, 4-, and 6- years of age). Children will be drawn from middle class pre-schools and primary schools.

Procedures. The subjects will be administered the gesture allowed and the gesture inhibited conditions described below in counterbalanced order. Set size will be counterbalanced within each condition.

Gesture allowed condition. The child will be presented with toy animals pasted on cards; six of the cards will contain 8 or 9 animals and six, 2 or 3 animals. As in the other tasks, in this proposal, the child will be told a story about the animals and will then be asked to count them.

Gesture inhibited condition. As above, except that the animals will be presented in one end of a box (the animals' cage) with a window at the

opposite end. In order for children to count the animals in this condition they must look into the box, and thus it will be impossible for them to use gestures to count.

Data analysis. The hypothesis is that although very young children use gestures when they count, they are not using these gestures to mediate their counting accuracy. However, with increasing age, gesture begins to serve a numerical function, and gradually gesture becomes an interiorized action such that children should not need to rely on overt gestures to achieve an accurate count.

This general model leads to the following prediction about the outcome of this study. At 2½ years of age, there will be no difference in children's counting accuracy across conditions. At four years there will be a marked difference, with the gesture allowed condition achieving a much higher level of accuracy than the gesture inhibited condition. At 6-years of age, there will be no difference between accuracy levels in the two conditions. An ANOVA (Age (3) x Condition (2) x Set size (2)) on children's error scores will be used to test this formulation.

Study 8

The purpose of this study is to determine age related changes in children's ability to assess the accuracy of their own counting in conditions which provide more or less possibility for an accurate count. In this study an interviewer will administer each child a set of three counting tasks. In one condition the child will be required to count objects piled in a beaker. In another condition the objects will be affixed in a random array to a flat surface. In the third condition the objects to be counted will be loose on a surface and the child will be able to manipulate them.

It is expected that a developmental pattern will be observed in children's ability to assess their own counting accuracy across conditions. We expect that young children (5-year-olds) will tend to believe that they have counted accurately across conditions. Over age children's ability to know when they have made errors should improve.

Subjects. A sample of 15 children at each of three age levels (5, 7, 9 year olds) will be interviewed. Each group will contain approximately the same number of boys and girls, and children will be drawn from middle class elementary schools in the New York area.

Procedures. The interviewer will conduct tasks with each child individually. The three tasks described will be administered at each of three set sizes (28, 29, 30) making a total of nine trials for each subject. Trials will be administered in a counterbalanced order within each age group. Prior to the administration of tasks, each child will be asked to count. If a child cannot count to 40, the child will be eliminated from the study.

Beaker task. Blocks or beans are contained in a plastic beaker. In this condition the child cannot touch or move the objects.

Fixed objects task. Blocks or beans are fixed to a piece of matboard in random array. In this condition the child can touch each block separately; however, they cannot be moved.

Loose objects task. Blocks or beans are presented in a loose pile. The child is able to manipulate objects freely.

Administration. With the interviewer and a child sitting on opposite sides of a table, the interviewer says, "I want to play some counting games with you. In some of the games the blocks or beans will be loose and you will be able to move them if you want. In other games they will be either stuck on a piece of cardboard or placed in a jar and you will not be able to move them."

At this point one of the trials is presented, and the child is told "count and tell me how many (blocks, beans) there are here." After the child responds and the number is recorded, the interviewer asks, "Are you sure there are (blocks, beans) or do you think you might be wrong?" If the child responds that he is sure he is right, the next trial is administered in the same fashion. If the child responds that he thinks he may be wrong, the interviewer asks the child "What is the biggest number of (blocks, beans) there could be?" What is the smallest number of (blocks, beans) there could be?" The answers to both of these questions are recorded.

Data analysis. Each child's confidence range will be calculated by subtracting the "least" judgments from the "most" judgments. A two-way ANOVA (age (3) x condition (3)) will be performed on these scores.

Study 9

The purpose of this study is to determine some of the conditions under which children's ability to assess their own counting accuracy improves. It is likely that the development of this ability is linked to children's experience of conflict in their counting activity — situations when they believe they have produced an

accurate count but discover, to the contrary, that they have not.

On the basis of pilot data, we have found that 5-year-olds generally believe that they count accurately in situations in which it is practically impossible for them to have achieved an accurate count (the beaker condition in the previous study), and that over age, children's ability to estimate their counting accuracy seems to improve. In this study we will provide subjects with different conflict conditions in order to determine whether, in fact, conflict does lead to greater ability to monitor counting accuracy.

Subjects. A sample of 90 children will be interviewed (45 4-year-olds and 45 6-year-olds) who are from middle class schools.

Procedures. All children will be assessed with the tasks described in Study 8. Children will then be matched for ability level within each age group and then assigned randomly to one of three conditions.

1. Condition I. The child is presented with modified task procedures from the beaker pretest condition and is asked, after each count, to count again (set size will be reduced from the pre-test). This is repeated for three sets of four trials with beakers of different shapes.

2. Condition II. The child is presented with modified task procedures from the beaker pretest condition and is told after each count what the actual number is. This is repeated over twelve different beakers.

3. Condition III. The pretest procedures will be re-administered for the beaker condition for twelve trials.

Posttest. Subjects will be posttested within a week following the training conditions. The posttest will be identical to the pretest. A follow-up posttest will also be conducted two months following the training conditions to assess whether training had a lasting effect.

Data analysis. Subjects' "change" scores will be determined from pre- to posttest for each condition by subtracting the posttest range from their pretest range. Two 3-way ANOVAs (age (2) x condition (3) x task-type (3)) will be used to determine whether conditions had an effect on children's monitoring of their counting accuracy for each of the two posttests.

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